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School segregation and the identification of tipping behavior☆

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ABSTRACT

We develop a method to empirically implement the Schelling model of segregation, and use it to study racial segregation in Los Angeles schools from 1995 to 2012. Our two-step method combines the estimation of parents' preferences for their children's peers with a counterfactual simulation analysis. We find substantial heterogeneity in the existence and locations of tipping points and stable equilibria. Schools are observed on equilibrium trajectories, but many remained out of equilibrium by 2012. We also introduce novel instrumental variables to identify preferences for endogenous peer groups that require no additional data and can be used in other educational settings.

1. Introduction

Birds of a feather flock together. Social forces draw similar peers to one another in diverse settings, often resulting in the segregation of individuals, which has important consequences for their behavior and well being. The Schelling model (Schelling, 1969, 1971) offers the seminal theoretical framework for the analysis of segregation in the social sciences. It has been used to study a variety of contexts, from racial segregation in neighborhoods and schools to gender segregation in occupations. In this paper, we develop a novel method to empirically implement the Schelling model, and we use it to study racial segregation among White, Black and Hispanic students in Los Angeles County public schools.

Dating at least as far back as the U.S. Supreme Court's landmark decision in *Brown v. Board of Education of Topeka* (1954), school segregation has occupied a prominent position in the public policy debate. Through the lens of the Schelling model, parents of different races respond differently to the prior observed racial compositions of schools when making enrollment decisions for their children. This may create a self-reinforcing, positive-feedback mechanism whereby the shares of minority students in schools change in each year as parents respond to the new racial composition. For example, if White parents have a stronger preference for White peers relative to minority parents, then there exists a threshold minority share above which a school will "tip" towards a stable equilibrium with a high share of minority students and below which a school will "tip" towards a stable equilibrium with a low share of minority students. This threshold is commonly referred to as a *tipping point*, and it represents an unstable equilibrium as even a slight perturbation around this point may lead to very different long-run racial compositions of the same school.

Despite considerable theoretical developments to the Schelling model (e.g. Becker and Murphy, 2000; Pancs and Vriend, 2007; Zhang, 2009), there have been few empirical developments in implementing this model to identify tipping behavior using observational data. An important reason for this disparity is the fact that the Schelling model obtains a concise explanation for segregation by assuming individual agents behave according to some plausible heuristics. Although the simplicity of this approach is appealing, it is not amenable to the traditional empirical tools that have been developed to identify preferences and equilibria in models of school and neighborhood choice beginning with McFadden (1973) (e.g., Bayer et al., 2007, 2004; Bayer and Timmins, 2005). However, these

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very empirical models of school and neighborhood choice may be inappropriate to analyze tipping behavior because they assume that households' choices are observed in equilibrium, i.e., in the absence of further shocks the racial composition of schools/neighborhoods will remain fixed. This assumption stands in contradiction to a central insight of the Schelling model that at any given point, schools and neighborhoods may be observed in the process of tipping – in disequilibrium, but converging to an equilibrium – rather than already having reached a stable, long run equilibrium.

Recently, Card et al. (2008a) have circumvented these issues with a reduced-form approach to identify tipping points in neighborhoods and schools as thresholds around which the flows of both Whites and minorities are qualitatively different. They argue that if the share of minority households (students) in a neighborhood (school) exceeds a tipping point, then it will experience relative outflows of White households (students), and vice versa.[1] However, the simplicity of this approach is not without cost, as any such reduced form identification strategy must assume that all schools or neighborhoods possess a common, fixed tipping point – an assumption that is generally invalid if schools or neighborhoods offer different or changing levels of amenities (Banzhaf and Walsh, 2013).[2] As a motivating example, we present the racial composition of two Los Angeles County schools over time in Fig. 1. Each school starts from a roughly equal composition of White and Hispanic students, but starting in 1990, their compositions evolve in different directions. Gardner Street Elementary becomes a predominantly White school while Fulbright Avenue Elementary becomes a predominantly Hispanic school. This disparity is inconsistent with the assumption that these two schools in the same city possess common tipping points.

In this paper, we provide empirical micro-foundations to the Schelling model by building upon the literature on neighborhood choice (e.g., Bayer and Timmins, 2005; Bayer and McMillan, 2010),[3] with the key difference that our method allows for individuals' choices to be observed out of long-run equilibrium. This is an empirically important feature, as the observed enrollment dynamics displayed in Fig. 1 are indicative of adjustment paths to new equilibria. Our empirical framework for studying tipping behavior offers four main innovations over existing approaches: First, we provide a method to infer race-specific preferences for the racial composition of peers from enrollment data. Second, we are able to identify all potential stable equilibria in addition to tipping points, which enables us to infer how far out of equilibrium the racial composition of each school is at any point in time. Third, our method allows for different schools to have different tipping points and stable equilibria. Schools with different characteristics, such as teachers, locations or funding levels, generally feature different tipping points and stable equilibria (Banzhaf and Walsh, 2013), so allowing for this heterogeneity lends credibility and realism to our estimates. Fourth, our method builds upon the discrete choice literature on neighborhood and school choice (e.g., Bayer and Timmins, 2005; Bayer et al., 2007; Bayer and McMillan, 2010), so it can flexibly accommodate many extensions to the Schelling model by easily borrowing tools from this well established literature.

Our framework is motivated by the idea that tipping behavior in a school can only be fully identified by analyzing the dynamic process of segregation from multiple initial states even though the racial composition of that school is only observed at a single initial state. This indicates a two-step structural approach that first estimates how the racial compositions of schools will evolve as a function of their initial states and subsequently simulates these trajectories from various counterfactual initial states. This allows us to uncover tipping behavior in the aggregate.

In the first step, we use school enrollment data to estimate separately White and minority parents' preferences for the racial composition of their children's schools.[4] This requires solving the widely known identification problem of isolating endogenous social effects from confounding effects (Manski, 1993), which we accomplish with a novel instrumental variables (IV) approach.

In the second step, we use these estimates to simulate the implied racial compositions of each school under different counterfactuals. For any counterfactual level of the share of minority students in a school in a given year, we compute the ensuing share of minority students that is implied under this counterfactual by allowing parents to re-sort holding all other school amenities constant. It is then straightforward to recover the unique tipping points and stable equilibria for each school in each year from the simulated schedules of their racial composition.[5] Unlike previous empirical approaches, our framework allows us to identify for each school and year the full trajectory of tipping behavior in the absence of external shocks, which can inform policies that aim to impact school segregation.

We perform our analysis on a sample of all students enrolled in public schools in Los Angeles County from 1995 to 2012 and find that race based tipping is a widespread and diverse phenomenon. Parents prefer peers of their own race, particularly in higher grades. Elementary schools rarely have tipping points, but most middle and high schools do with their locations ranging from a minority share of 20% to a minority share of 80% depending on the school's characteristics. All stable equilibria are highly segregated; one group of equilibria range from 0% to 20% minority, and another group of equilibria ranges from 80% to 100% minority. In high schools, social forces are so strong that two otherwise similar schools may converge to equilibria that differ by up to 100 percentage points. The minority shares of schools have moved closer to a stable equilibrium during the sample period; however, many schools had not yet converged to a stable equilibrium by 2012: about 20% of schools were observed out of equilibrium by at least 20 percentage points of their minority share of enrollment.

We extend our analysis by allowing parents to have heterogeneous preferences for Black and Hispanic peers. This more complex and realistic model implies higher-dimensional dynamic behavior and enables us to distinguish potential tipping to segregated Black equilibria from potential tipping to segregated Hispanic equilibria, even within the same school. Overall, we find that our baseline results provide an accurate, if simplified, portrayal of school segregation in LA County.

1 Pryor (1971) conducts a similar empirical exercise using a related approach.

2 Easterly (2009) takes an alternative reduced form approach to identify tipping behavior that also relies on the assumption that all neighborhoods possess a common, fixed tipping point.

3 As discussed in Bayer and Timmins (2005) and Bayer and Timmins (2007), one important difference between the neighborhood choice literature and the discrete choice social interaction literature (e.g., Brock and Durlauf, 2001a; Brock and Durlauf, 2001b; Blume and Durlauf, 2003) is that the former literature explicitly accounts for unobservable school amenities that are correlated across individuals, which are known to be empirically relevant to location choice (Bayer et al., 2007; Caetano, 2016). Because of this difference, these literatures generally apply to distinct economic environments. For instance, in the discrete choice social interaction literature, the number of choices is fixed and is often binary, whereas in the literature on school and neighborhood choice, asymptotic results require the number of options to be large.

4 Ioannides and Zabel (2008) estimate households' preferences for a variety of other social amenities in a nested model of neighborhood and house choice.

5 Bayer and Timmins (2005) present a different simulation technique to identify multiple equilibria in the context of social interactions under the assumption that choices are observed in equilibrium. Bayer and McMillan (2010) estimate an equilibrium model of school choice and provide a simulation technique to estimate measures of school competition, but they do not consider social interactions. In a computational study of residential segregation, Bruch and Mare (2006) simulate flows of White and minority residents between neighborhoods under a variety of assumptions, but they do not empirically identify tipping points or stable equilibria.

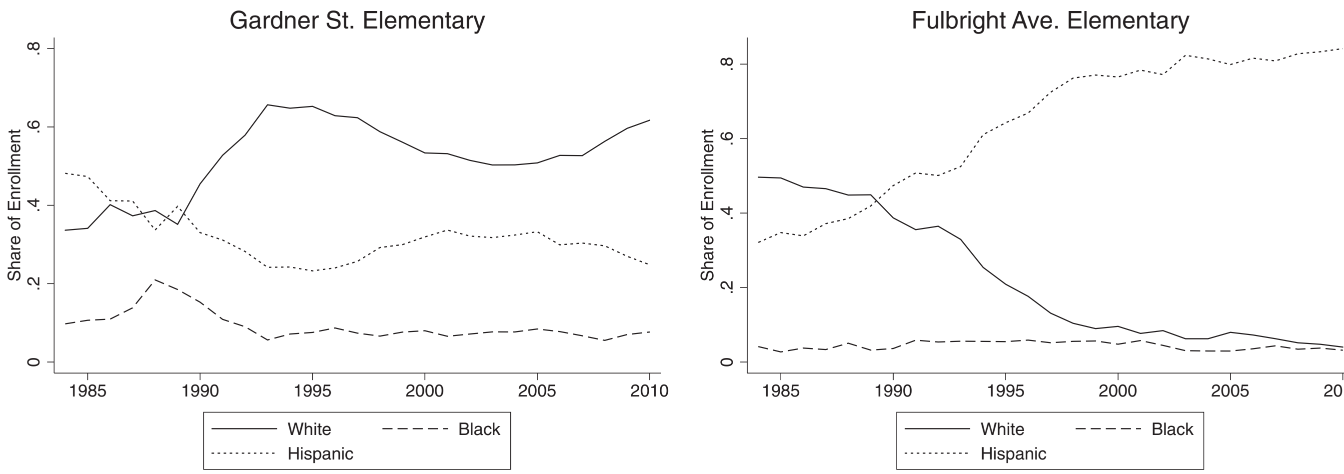

Fig. 1. Racial composition of enrollment for two selected schools, 1984–2010.
Source: Common Core of Data, US Department of Education.

Racial segregation has been studied predominantly in schools and neighborhoods. We implement our empirical method in the context of school segregation for three reasons. First, our IV exploits the fact that school enrollments are stratified by grade. Second, the timing of the school decision (annually) is clearer. In the case of neighborhood choice, for example, households moving at the beginning of a calendar year may observe different minority shares than households moving at the end of the year. Third, the analysis of tipping requires relatively high frequency data. Public schools are required by federal law to report the racial composition of their enrollment annually, whereas the racial composition of neighborhoods is only available at a lower frequency. Thus, we follow a long empirical and theoretical literature that has analyzed the causes (Echenique et al., 2006; Frankel and Volij, 2011) and effects of school segregation (Jackson, 2009; Boustan, 2012; Billings et al., 2012) and has established school segregation as an important topic in its own right. We do provide suggestive empirical evidence that school segregation is a distinct phenomenon that is occurring over and above any underlying neighborhood segregation, but a satisfactory joint analysis of school and neighborhood segregation is beyond the scope of this paper.

The remainder of the paper is organized as follows. In Section 2, we briefly introduce the Schelling model of segregation to highlight the inherent challenges in identifying tipping points and stable equilibria. In Section 3, we present a two stage empirical strategy that identifies tipping behavior through estimation and simulation, and in Section 4, we propose a novel instrumental variables approach to facilitate estimation. In Section 5, we describe our data set and present our main empirical results. In Section 6, we interpret our empirical findings in the context of the Schelling model. In Section 7, we extend our analysis further by considering higher dimensional tipping between White, Black and Hispanic students. We conclude by highlighting some directions for future research.

2. Identification of tipping points and stable equilibria

The seminal Schelling (1969) model of segregation and its successors share two key features that have important theoretical and empirical implications for tipping behavior. First, for tipping to occur, groups must have different preferences for the racial compositions of schools.[6] This difference in preferences is necessary but insufficient to generate tipping behavior. Second, since tipping is characterized as a dynamic adjustment process, there must exist some friction that ensures that agents do not always immediately take equilibrium actions. In the Schelling model, this friction arises because agents are cast as myopic decision makers.[7]

The Schelling model differs from many standard economic models in that it describes the aggregate phenomenon of segregation through a framework in which individual agents make decisions according to simple heuristics. Becker and Murphy (2000) offer an alternative representation of the model that is instead based upon the standard economic primitives of preferences for peer groups, which we use to motivate the empirical challenges in identifying tipping points and stable equilibria.

Suppose there are two groups of parents indexed by r, where $r = W$ if the parent is White and $r = M$ if the parent is a minority. At the beginning of each school year, parents choose a school for their child to attend. Parents observe a set of amenities for each school j: an endogenous social amenity s_j, which represents the minority share in the school, and an exogenous vector of other amenities X_j, which may include such factors as other characteristics of the school, the (implicit) price of attending the school, and characteristics of competing schools. Parents are assumed to be myopic; that is, they observe each amenity at its level at the end of the previous school year and select their school for the upcoming year without taking into account the simultaneous decisions of other parents. Aggregate parental demand functions can be written as $n_j^r(s,X)$, which represent the total number of parents of race r who demand to send their child to school j. The resulting minority share in school j in the next school year will be

$$S_j\left(s_j,X_j\right) = \frac{n_j^M\left(s_j,X_j\right)}{n_j^W\left(s_j,X_j\right) + n_j^M\left(s_j,X_j\right)}. \quad (1)$$

It is important to note that the function in Eq. (1) has a causal interpretation: for a given X_j, changes in s_j change the demands of Whites and minorities, which in turn change S_j.

Fig. 2 illustrates a theoretical plot of $S_j(s,X_j)$ for particular demand curves $n_j^W(s,X_j)$ and $n_j^M(s,X_j)$. Values of s where the curve crosses the

[6] Zhang (2009) generalizes Schelling's model and shows that even when individuals have a preference for integration in the aggregate, a slight difference in the preferences of two groups can still lead to fully segregated equilibria.

[7] Myopia is assumed in several spatial models of learning in urban economics and economic geography (e.g., Maskell and Malmberg, 2007). Kandori et al. (1993) justify myopia in models of social interactions if agents have difficulty conceptualizing the best responses of others. Levinthal and March (1993) provide an overview of the theoretical and empirical literature on myopia in learning.

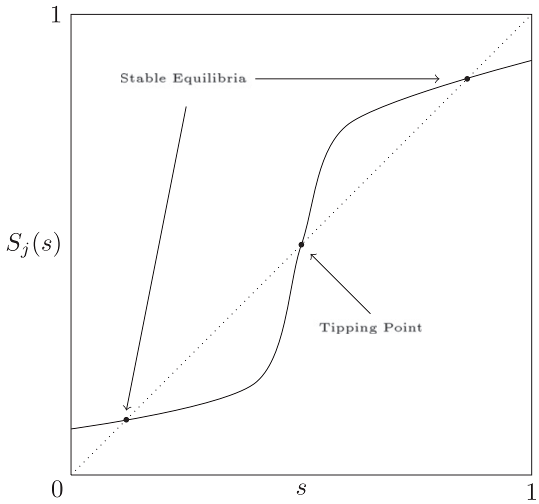

Fig. 2. Theoretical illustration. Note: This figure depicts a curve representing the resulting minority share in school j (S_j) that is implied by the parental response to different values of the prior minority share (s).

45-degree line (i.e., $S_j(s, X_j) = s$) are equilibria; for these values of s, the minority share of students at the school is not expected to change in the next period in the absence of shocks. A tipping point, or unstable equilibrium, is a point that crosses the 45-degree line from below, and a stable equilibrium is a point that crosses the 45-degree line from above.[8] At a stable equilibrium, small deviations of s will result in Whites and minorities re-sorting in such a way that the minority share will return to the stable equilibrium level. At a tipping point, small deviations of s will result in Whites and minorities re-sorting in such a way that the minority share will diverge from the tipping point towards a stable equilibrium.

Empirical identification of tipping points and stable equilibria is complicated by the fact that the demand schedules of the groups may be difficult to recover. The identification is further complicated if parents face a multinomial choice rather than a binary choice, as X_j will include not only school j amenities but also the amenities of other schools (including the share of minority students in these schools). However, Fig. 2 suggests a natural reduced-form approach to identify tipping points without the specification of all relevant demand functions, which has been implemented by Pryor (1971), Card et al. (2008a) and Pan (2015). We describe this approach, discuss its drawbacks and then propose our alternative identification strategy.

Suppose s_j is observed for two periods, t and $t+1$, in the sample of all schools in a given city, all of which share a common tipping point. One could plot s_{jt+1} on s_{jt} for all j on a single set of axes as in Fig. 2. Tipping points could be identified as values of s_{jt} for which the plotted curve crosses the 45 degree line from below.

This identification strategy relies on two strong assumptions. First, all schools in the sample must share a common tipping point at period t. To see this, note that the entire sample of schools in the city is used in order to identify a single point. But to the extent that schools offer different levels of private amenities to their students (i.e., $X_j \neq X_k$), the demand schedules of parents for different schools are not generically the same, so in general this assumption will be invalid. The second assumption underlying this reduced-form identification strategy is that all amenities in X_j must remain fixed from periods t to $t+1$ since each point in the plot is an ordered pair (s_{jt}, s_{jt+1}) that is assumed to reflect an identical school in all respects besides minority share. But $X_{jt} \neq X_{jt+1}$ in general, so tipping points and stable equilibria are likely to change from t to $t+1$. This complicates the interpretation of any fixed point approach that equates shares of minority students in periods t and $t+1$.

The key empirical takeaway from this discussion is that heterogeneity in tipping behavior is derived from heterogeneity in the levels of private amenities in the schools as opposed to heterogeneity in parental preferences (Banzhaf and Walsh, 2013).[9] Hence, all approaches to identify tipping behavior that implicitly assume a common tipping point across any subset of schools must also assume that those schools possess identical, fixed levels of private amenities; otherwise it may be difficult to attribute the findings to an underlying tipping mechanism.

In order to avoid these assumptions, we develop a structural approach to recover tipping behavior in which we essentially construct Fig. 2 separately for each school and each year.

3. An empirical approach to identify tipping behavior

Our empirical approach has two steps, which can be summarized as follows: First, we estimate separate demand schedules $n_{jt}^W(s)$ and $n_{jt}^M(s)$ for each school j in each year t. Importantly, we allow Whites and minorities to have different preferences for the racial composition of schools. We use instrumental variables based on inter-cohort variation in enrollments to identify the causal effects of s on the demands of parents of each race and grade level. With estimates of these causal effects, we can simulate $S_{jt}(s)$ as a *ceteris paribus* function of any counterfactual value of the share of minority students in that school in a given year, allowing parents to re-sort across all schools and holding X_{jt} constant. That is, we construct S_{jt} by simulating movements *along* parents' demand schedules. The simulated function $S_{jt}(s)$ should be interpreted as the minority share that is implied by the counterfactual s in the absence of any shocks. Having identified the entire curve $S_{jt}(s)$, it is straightforward to recover tipping points and stable equilibria for each school in each year.

3.1. Step 1: Estimating parental preferences for minority peers

In year t, n_{jt}^{gr} children of race r enroll in one of the public schools that offer instruction in grade g in LA County. The minority share of students at school j is given by

$$s_{jt} = \frac{\sum\limits_{g \in G_j} n_{jt}^{gM}}{\sum\limits_{g \in G_j} \left(n_{jt}^{gM} + n_{jt}^{gW} \right)} \tag{2}$$

where $G_j = \left\{ \underline{g}_j, \ldots, \bar{g}_j \right\}$ is the range of grades for which school j offers instruction. Parents enroll their children in year t having observed school amenities at the end of year $t-1$. In accordance with the Schelling model, parents do not strategically extrapolate

[8] Points at which the curve S_j crosses the 45-degree line from above with a negative slope are not necessarily stable equilibria. For values of s around these points, we will observe oscillating dynamics that can lead to either convergence towards the crossing point or divergence from it depending on the steepness of S_j. As we do not observe these more complex dynamics in our baseline empirical analysis, we ignore them for simplicity.

[9] It is difficult to interpret the discontinuities found in Card et al. (2008a) as tipping points for two reasons. As the running variable (minority share in the prior period) changes, the unknown point of discontinuity will likely change with it, complicating the interpretation of discontinuities as tipping points. Moreover, in a deviation from standard Schelling models, Card et al. (2008a) define tipping points as any level of s for which the curve S_j is *discontinuous* (as recognized by Card et al., 2008a, these are usually referred to as bifurcation points) in order to employ regression discontinuity methods to identify structural breaks.

other parents' future enrollment decisions when making their own, so dynamic adjustment unfolds at a year-by-year pace.[10]

Parental demand is written as

$$\log n_{jt}^{gr} = \beta^{gr} s_{jt-1} + u_{jt}^{gr} \tag{3}$$

where the parameters β^{gr} represent the preferences for the racial composition of schools and u_{jt}^{gr} is an error term.[11]

The two terms on the right hand side of Eq. (3) correspond to distinct sources of dynamics in school choice. The first term describes an explicit endogenous relationship between the prior minority share in schools and the current race-specific demands for schooling. This relationship is often referred to as the *endogenous social effect* and is precisely the source of dynamics that is prescribed by the Schelling model. It represents the response of parents to the prior minority share of the school *per se*. The second term accounts for all other determinants of grade and race specific demands. To identify β^{gr}, we need to isolate the response of parents to the prior minority share *per se* from their response to all other determinants that might be correlated to the prior minority share. We do so with a novel instrumental variables approach that we outline below.

Consider a school that offers instruction in grades $g \in \{\underline{g}_j, \ldots, \bar{g}_j\}$. We modify Eq. (3) as

$$\log n_{jt}^{gr} = \beta^{gr} s_{jt-1} + \underbrace{\sum_{i=\underline{g}_j}^{\bar{g}_j-1} \left(\alpha_{iW}^{gr} \log n_{jt-1}^{iW} + \alpha_{iM}^{gr} \log n_{jt-1}^{iM}\right)}_{C_{jt-1}^{gr}} + \epsilon_{jt}^{gr}, \tag{4}$$

where C_{jt-1}^{gr} represents the year $t-1$ log-enrollments of all races for all grades of the school, except the last grade. Let $s_{jt-2}^{\bar{g}_j-1} = n_{jt-2}^{\bar{g}_j-1M} \Big/ \left(n_{jt-2}^{\bar{g}_j-1W} + n_{jt-2}^{\bar{g}_j-1M}\right)$ be the minority share of students in grade $\bar{g}_j-1$ in school j in year $t-2$. Our identifying assumption is that $s_{jt-2}^{\bar{g}_j-1}$ is an instrumental variable for s_{jt-1}, conditional on C_{jt-1}^{gr}:

Assumption 1. *Identifying assumption.* $\mathrm{Cov}\left[s_{jt-2}^{\bar{g}_j-1}, \epsilon_{jt}^{gr} | C_{jt-1}^{gr}\right] = 0$.[12]

Intuitively, $s_{jt-2}^{\bar{g}_j-1}$ is determined by school amenities in $t-2$ that either do or do not persist to t. The persistent amenities are endogenous, so we must control for them; the transitory amenities are exogenous and constitute clean identifying variation. By adding C_{jt-1}^{gr} as controls, we absorb the persistent amenities without absorbing the transitory ones. Our identifying assumption is that amenities that do not persist from $t-2$ to $t-1$ do not suddenly become relevant again in t. Under Assumption 1, β^{gr} is consistently estimated in Eq. (4) by 2SLS.[13]

3.2. Step 2: Recovering tipping points and stable equilibria

In the second step, the estimates of β^{gr} are used to simulate the implied minority share function $S_{jt}(s)$.[14] We simulate $S_{jt}(s)$ in the absence of shocks from t to $t+1$, which corresponds to an environment in which fixed points of $S_j(s)$ correspond to tipping points and stable equilibria. This procedure allows us to identify tipping points and stable equilibria for each j and t even though school amenities are not necessarily fixed over time or common to schools. We first construct the counterfactual demand function

$$n_{jt}^{gr}(s) = \exp\left(\log\left(n_{jt}^{gr}\right) + \hat{\beta}^{gr}\left(s - s_{jt-1}\right)\right) \tag{5}$$

which represents the expected number of race r students that would enroll in grade g in school j in year t for a given counterfactual value of $s_{jt-1} = s$.[15] In order to ensure that the total student population of LA County remains constant for each level of the counterfactual minority share, we rescale Eq. (5) as follows[16]:

$$\tilde{n}_{jt}^{gr}(s) = \frac{n_{jt}^{gr}(s)}{n_{jt}^{gr}(s) + \sum\limits_{k \neq j} n_{kt}^{gr}} \cdot \sum_k n_{kt}^{gr} \tag{6}$$

The implied share of minority students is then defined as

$$S_{jt}(s) = \frac{\sum\limits_{g=\underline{g}_j}^{\bar{g}_j} \tilde{n}_{jt}^{gM}(s)}{\sum\limits_{g=\underline{g}_j}^{\bar{g}_j} \left(\tilde{n}_{jt}^{M}(s) + \tilde{n}_{jt}^{W}(s)\right)} \tag{7}$$

The numerator of Eq. (7) is the number of minority students that would enroll in school j if its minority share was previously s, and the denominator is the total enrollment of school j if its minority share was previously s. A plot of S_{jt} on s is the empirical analog to Fig. 2. Each point of the simulated curve $S_{jt}(s)$ corresponds to the implied minority share for school j at time t under the counterfactual assumption that $s_{jt-1} = s$. Note that the only source of dynamics that we use to identify S_{jt} is the endogenous social effect, since our simulation is performed in the absence of other sources of dynamics (i.e., we hold u_{jt}^{gr} fixed).

In period t, school j possesses either a tipping point or a stable equilibrium at any level of s where $S_{jt}(s) = s$. This equation does not generally possess an analytical solution, so we use a simple numerical technique to estimate tipping points and stable equilibria. We allow s to take on values ranging from 0 to 1 in increments of 0.01, and at each value of s, we simulate S_{jt} using Eq. (7). We then plot these simulated shares S_{jt} on s and locate the value(s) of s for which the plot crosses the 45 degree line. A value of s for which the simulated function S_{jt} crosses the 45-degree line from below (i.e., $S_{jt}' > 1$) represents a tipping point $s^\star$, and a value of s for which it crosses

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[10] This assumption serves the additional purpose of helping us break the simultaneity problem present in equilibrium choice models with social interactions (Manski, 1993) since s_{jt-1} is not a function of n_{jt}^{gr}, but s_{jt} is. We find strong support for this assumption in Section 5.

[11] We specify the racial composition of the school instead of the racial composition of a particular grade as the main explanatory variable for two reasons. First, it is easier for parents to observe the racial composition of the school than of a particular grade. This is true whether they look at school report cards or whether they visit the school and physically observe the student body. Second, parents may care about social interactions that happen across grades (e.g., an atmosphere of bullying).

[12] This assumption contains an abuse of notation in order to simplify the exposition. We condition on the variables in $\left\{\log n_{jt-1}^{gr}; g = \underline{g}_j, \ldots, \bar{g}_j - 1, r = W, M\right\}$, not on C_{jt-1}^{gr} as written above. In practice, we find that a linear projection of these variables and a more flexible specification of these variables generate the same results.

[13] An IV that follows this property is often called a "Conditional IV". See Angrist and Pischke (2009, pp. 175).

[14] To simplify the notation, we refer to $S_{jt}(s, X_{jt-1})$ as $S_{jt}(s)$ hereafter. This is without loss of generality as X_{jt-1} is held constant in the simulation.

[15] Note that the variable n_{jt}^{gr} represents the *actual* number of students in the data, while the function $n_{jt}^{gr}(s)$ represents the *implied* number of students for a counterfactual level of the prior minority share. Hats correspond to estimated parameters. Taking logs of both sides of Eq. (5) hence yields the *implied* empirical analog of parental demand as written in Eq. (3) under the assumption that $u_{jt}^{gr} = 0$.

[16] In practice, the rescaling in Eq. (6) is unimportant $\left(\tilde{n}_{jt}^{gr}(s) \approx n_{jt}^{gr}(s)\right)$ because of the large number of public schools in LA County. We do it primarily to maintain the equivalence between the estimation approach described in the main text and the discrete choice estimation approach described in the appendix.

the 45-degree line from above (i.e., $S'_{jt} < 1$) represents a stable equilibrium $s^{\star\star}$.[17]

4. Instrumental variables

In this section, we discuss the intuition behind the IV that we use in the first step of our empirical approach.

Consider a grade 9–12 high school with an outstanding band director who retired three years ago. Her past work may still indirectly affect new enrollments today through the enrollments of students that had the privilege of her instruction. The band director affected the enrollments of 9th graders in $t-4$ who eventually became 12th graders in $t-1$. If the band director was valued differently by White and minority parents, then her past presence would affect s_{jt-1}, which in turn would affect student enrollments in t. This effect is entirely transmitted through the single cohort of remaining students that she influenced – 12th graders in $t-1$. Because this cohort of students will not be enrolled in t– they have aged out – any remaining effect of the band director on student enrollments in t is plausibly exogenous.

Of course, this band director is a stylized example, and we cannot observe a comprehensive list of specific past amenities that no longer remain. We circumvent this issue with an identification strategy that relies only on enrollment data at the grade-year level. We explain our strategy as follows: First, note that the 11th grade students in $t-2$ do not attend the school in t because they have aged out. This allows us to use the minority share of this cohort of students, the *IV cohort*, as an instrument since these students do not show up in the dependent variable. Second, note that the amenities that led students in the IV cohort to enroll in $t-2$ may or may not be present in t. In order to absorb any confounding amenities that persisted to t, we use the enrollments of the younger *control cohorts* in $t-1$ (when they are in grades 9–11) as control variables. By doing so, our instrument is comprised of only those amenities that caused the IV cohort to enroll in the past and did not directly cause any subsequent cohort to enroll.

We summarize our strategy for the example of the 9–12 high school in the diagram below. Our IV is s^{11}_{jt-2}, the minority share of the second highest grade in $t-2$. For our IV to be valid, we control for the $t-1$ enrollments of Whites and minorities in all grades except for the highest grade (i.e., grades 9, 10 and 11). Because we cannot control for the $t-1$ enrollments of Whites and minorities in the highest grade (since no variation in s_{jt-1} would remain), we use the IV cohort's enrollments in $t-2$ to ensure we are not identified off of persistent amenities specific to the highest grade.

	9th	10th	11th	12th
t	Dep. var.	Dep. var.	Dep. var.	Dep. var.
$t-1$	Control	Control	Control	
$t-2$			IV	

This logic is valid for any school that offers instruction for at least two grades, irrespective of its grade range. For any school that offers more than two grades of instruction, we can construct additional IVs from the minority shares of the third highest grade in $t-3$, or the fourth highest grade in $t-4$, etc.

4.1. Is the identifying assumption satisfied?

Assumption 1 fails to hold only if there exist unobservables that satisfy the following three properties:

1. They affect enrollment decisions in t (i.e., they are included in ϵ^{gr}_{jt} in Eq. (4)).
2. They are correlated to the minority share of grade $\bar{g}_j-1$ in $t-2$ (i.e., they are correlated to our IV).
3. They are not correlated to enrollment decisions of any race in any of the grades $\underline{g}_j,\ldots,\bar{g}_j-1$ in $t-1$ (i.e., they are not absorbed by C^{gr}_{jt-1}).

To justify our identification strategy, we consider an unobservable that satisfies the first two properties and then argue that it does not plausibly satisfy the third property. This unobservable is an amenity that is either unique to grade $\bar{g}_j$ in $t-1$, or it is an amenity that is not unique to grade $\bar{g}_j$ in $t-1$.

Any amenity that is unique to grade $\bar{g}_j$ in $t-1$ will fail to satisfy property 3 if students in some grade $g<\bar{g}_j$ in $t-1$ *anticipate* the amenity will be present in t. This is likely because the amenity must be present in t (property 1), and must be anticipated by students of different races in grade $\bar{g}_j-1$ in $t-2$ (property 2).

Any amenity that is not unique to grade $\bar{g}_j$ in $t-1$ is valued by at least some students enrolled in some grade $g<\bar{g}_j$ in $t-1$. As a result, it will fail to satisfy property 3. For instance, imagine that a 9–12 high school features a good library in t (property 1), and that the library is valued in $t-2$ by 11th grade students (property 2). As long as the library is valued by students in any of the control cohorts (i.e., students of any race in grades 9, 10 or 11 in $t-1$), property 3 will fail to hold.

There are a few potential issues with our identification strategy. One important consideration is whether a log-linear specification such as C^{gr}_{jt-1} fully absorbs the enrollments of students in the control cohorts. A second potential consideration is that students in the IV cohort might systematically repeat the last grade of the school. A third potential consideration is whether students in the first grade of the school in t (who are not directly controlled for by C^{gr}_{jt-1} because they did not attend that school in $t-1$) pose any threat to our identification. Finally, a fourth potential consideration is that students of the IV cohort may have younger siblings who enter the same school in t.

We empirically address all these potential concerns with robustness checks in Appendix B. We conduct falsification tests because we have multiple IVs for most schools in our sample since they offer more than two grades of instruction: $s^{\bar{g}_j-1}_{jt-2}, s^{\bar{g}_j-2}_{jt-3}, s^{\bar{g}_j-3}_{jt-4},\ldots$ Because these alternative IVs contain the same cohort specific variation but different grade specific variation, a comparison of the different IV estimates amounts to a test of whether C^{gr}_{jt-1} controls for the (potentially confounding) grade specific variation component of these IVs. The availability of additional IVs also allows for a formal over-identification test (Hansen, 1982) of Assumption 1. The results of all of these tests and robustness checks strongly support our identification strategy.

5. Data and results

5.1. Sample

We construct a sample of every public school in Los Angeles County that offered instruction in any grade from kindergarten through 12th

[17] Schools that are observed at off-equilibrium points will not necessarily converge over time to their corresponding stable equilibrium because the S curve may change over time. For each $\tau=1,2,\ldots$, the curve may change for either of the following two reasons: First, there may be future school-level shocks $\left(\mu^{gr}_{jt+\tau}\right)$, which will imply an exogenous shift in the curve at $t+\tau$. Second, as parents re-sort from off-equilibrium points in other schools from $t+\tau-1$ to $t+\tau$ due to tipping, the curve of school j will shift endogenously in response. As such, our simulation procedure should be interpreted as providing a snapshot of the S curve of each school at period t. Because of the large number of schools in our sample, this computationally simple partial equilibrium analysis is plausibly valid. In principle, our simulation procedure can be modified to account for the endogenous shifts in a general equilibrium analysis.

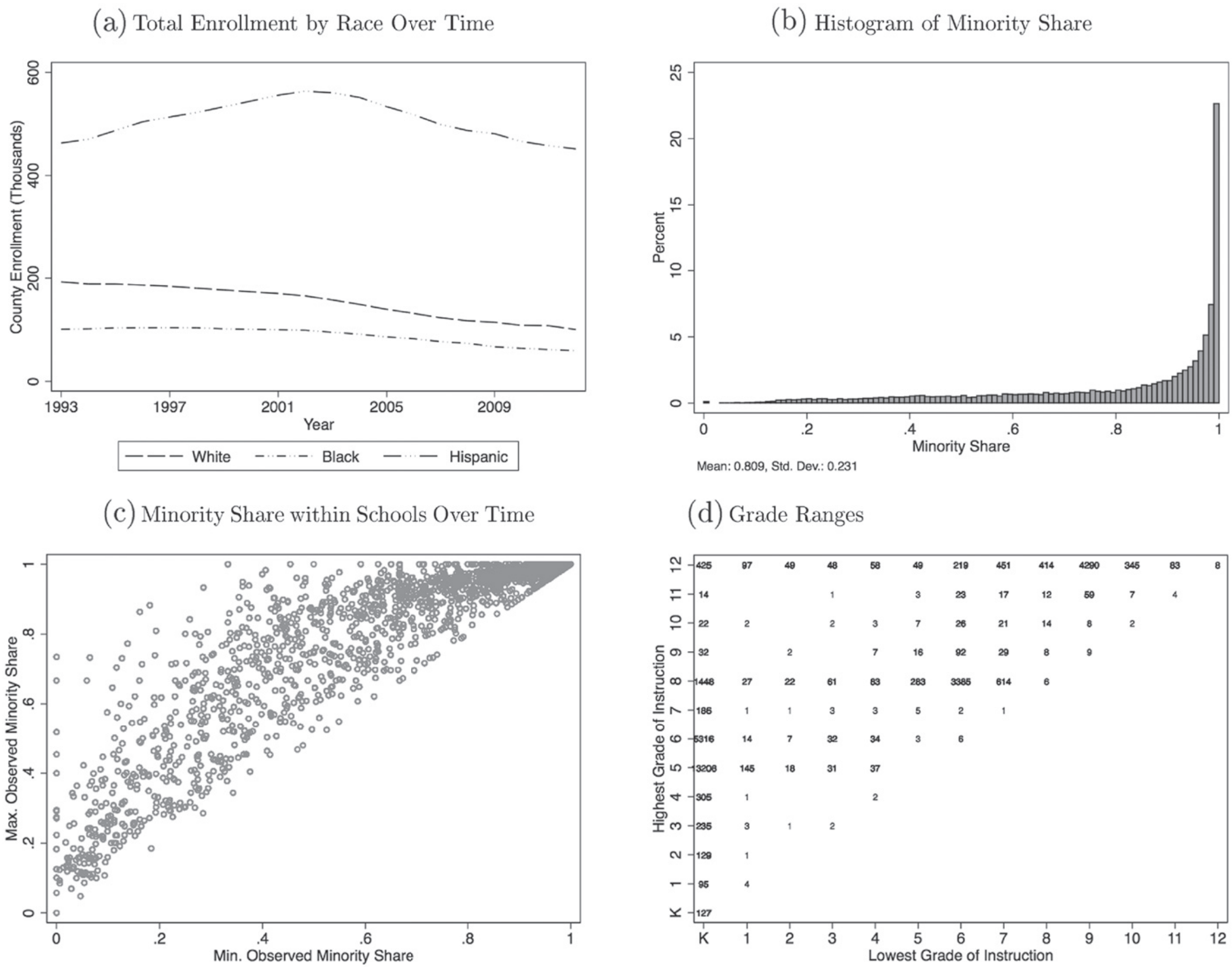

Fig. 3. Enrollment, racial composition and instruction in Los Angeles county schools, 1993–2012. (a) Total enrollment by race over time. (b) Histogram of minority share. (c) Minority share within schools over time. (d) Grade ranges. Note: Panel (b) shows a histogram of the minority shares in all school-years of the sample. In Panel (c), each dot represents a school. For each school, we present the minimum and maximum minority shares observed during the sample period. In Panel (d), each cell corresponds to the number of school-year observations in our sample for a given grade range. For example, we observe 4290 school-year combinations that offer instruction from grades 9 to 12.
Source: California Department of Education.

grade for all years between 1993 and 2012.[18] For each of the 2196 schools in the sample, we obtain grade and race level enrollment statistics from the California Basic Educational Data System (CBEDS) maintained by the California Department of Education.[19]

Over the sample period, LA County public schools experienced substantial changes in the aggregate racial composition of enrollments, which we present in the first panel of Fig. 3. Despite our terminology, the number of minority students enrolled in LA County public schools greatly exceeded the number of (non-Hispanic) White students in all years. A small absolute decline in White and Black enrollment was accompanied by substantial Hispanic inflows into the public school system until 2003.[20] Although a Public School Choice (PSC) resolution was adopted by LAUSD, the predominant school district in LA County, prior to the 2009–2010 academic year, fewer than 3% of students exercised school choice through the program as of 2012.[21] In addition, the first charter school in LA Country was approved in 1993 programs, and over 200 charters were operating in LA Country in 2012. These changes to the public school environment during our sample period, both demographic and institutional, underscore the importance of the use of time-varying (and race-, neighborhood- and grade-varying) fixed effects in our analysis.

18 For our purposes, "year" refers to academic year by registration date, not calendar year. For example, 2012 corresponds to the Fall 2012–Spring 2013 academic year.

19 Individual level student data for LA County is unavailable in restricted or unrestricted formats.

20 Enrollment trends are qualitatively similar across grades. The observed decline in White enrollment is probably due to declining fertility rates, as total private school enrollment in Los Angeles County decreased from roughly 220,000 students in 1999 to 160,000 students in 2012. Private school enrollment data prior to 1998 is not publicly maintained. (Source: CBEDS data collection, 1999–2000 and 2012–2013 Private School Affidavits.)

21 Source: LAUSD Board of Education website.

Table 1
Parental demand for schooling, 1995–2012: Baseline parameter estimates.

	(1)		(2)	
	OLS		2SLS	
r:	W	M	W	M
β^{Kr}	−6.33**	3.34**	−4.40**	1.71**
	(0.21)	(0.13)	(0.27)	(0.14)
β^{1r}	−6.76**	3.27**	−2.50**	1.24**
	(0.23)	(0.13)	(0.24)	(0.13)
β^{2r}	−6.51**	3.20**	−2.26**	0.89**
	(0.22)	(0.14)	(0.30)	(0.15)
β^{3r}	−6.76**	3.22	−2.55**	0.90**
	(0.24)	(0.16)	(0.23)	(0.15)
β^{4r}	−6.84**	3.08**	−2.49**	1.00**
	(0.25)	(0.16)	(0.24)	(0.13)
β^{5r}	−6.76**	3.00**	−2.36**	1.35**
	(0.24)	(0.16)	(0.24)	(0.15)
β^{6r}	−5.04**	3.86**	−2.96**	2.70**
	(0.44)	(0.35)	(0.47)	(0.36)
β^{7r}	−4.40**	4.33**	−3.15**	3.22**
	(0.65)	(0.58)	(0.76)	(0.66)
β^{8r}	−5.71**	4.53**	−2.78**	4.22**
	(0.65)	(0.56)	(0.81)	(0.58)
β^{9r}	−12.24**	−0.78	−4.56**	0.05
	(1.03)	(0.71)	(1.06)	(0.68)
β^{10r}	−12.34**	−1.22*	−4.74**	0.32
	(0.83)	(0.60)	(0.90)	(0.63)
β^{11r}	−9.62**	0.65	−3.98**	2.23**
	(0.69)	(0.52)	(0.81)	(0.53)
β^{12r}	−9.72**	0.44	−4.62**	1.50**
	(0.67)	(0.40)	(0.86)	(0.55)
Controls?	No		Yes	
IVs?	–		$s_{t-2}^{\bar{g}_j-1}$	
Grade-race-year-ZIP FEs	Yes		Yes	
R^2	0.56		0.43	
Num. obs.	362,516		361,866	

Notes: The dependent variable is log enrollment by grade, race, school and year $\left(\log n_{jt}^{gr}\right)$. Specification (2) includes $\log n_{jt-1}^{\underline{g}_j W}, \ldots, \log n_{jt-1}^{\bar{g}_j-1W}$ and $\log n_{jt-1}^{\underline{g}_j M}, \ldots, \log n_{jt-1}^{\bar{g}_j-1M}$ $\left(\text{i.e., } C_{jt-1}^{gr}\right)$ as controls. Robust standard errors clustered by grade, race, year and ZIP code are provided in parentheses.

* Statistically significant at the 95% level.

** Statistically significant at the 99% level.

At the school level, we observe substantial cross-sectional and longitudinal variation in minority shares. In the second panel of Fig. 3, we present a histogram of s_{jt} for all schools and years in our sample. The average school in our sample has a minority share of roughly 80% with a sizable standard deviation of roughly 23%. In the third panel of this figure, we present a scatter plot of the minimum and maximum observed values of s_{jt} for each j, which reflects the longitudinal variation in minority shares within schools. Each point corresponds to a single school, and the distance of each point from the 45-degree line represents the extent to which the minority share of that school varies over time. The observed aggregate longitudinal evidence implies that minority shares will tend to increase in all schools in the absence of school specific shocks, while the observed cross-sectional evidence hints at variation in amenities across schools. The within-school variation in s_{jt} facilitates estimation of the parameters and especially the simulation of S_{jt}.

Finally, in the fourth panel of Fig. 3, we document the substantial heterogeneity in the grades of instruction offered during our sample period. Most, but not all, of this variation comes from heterogeneity across schools. Although certain grade ranges are more commonly observed (e.g., K–5, 6–8 and 9–12), there are still sizable numbers of schools that offer instruction in other grade ranges during the sample period. The longitudinal and cross-sectional variation in enrollments combined with the heterogeneity in grade ranges of instruction all contribute to our identification strategy.

5.2. Estimation results

In Table 1, we present OLS and IV estimates of parental preference parameters for the racial composition of schools, β^{gr}.[22] In both specifications, we include grade-race-year-ZIP code fixed effects at the five digit ZIP code level.[23] By doing so, we account for the empirical fact that the set of students in LA County public schools is changing over time, and we allow for parents to value neighborhood amenities (including residential racial composition) at the ZIP code level differently on average. As such, we identify β^{gr} only off of variation in the minority shares of schools within the same ZIP code that instruct at least one common grade. (This subsample amounts to over 98% of the sample and well over 99% of the aggregate public school enrollment in LA County.) Standard errors are clustered by grade, race, year and ZIP code to account for the potential serial correlation of unobserved amenities within neighborhoods.

In the first specification of Table 1, we report naive OLS estimates of β^{gr} (we exclude C_{jt-1}^{gr} from this specification). We find that Whites possess relatively strong preferences against enrolling their children in schools with higher minority shares $\left(\hat{\beta}^{gW} < 0\right)$, whereas minorities possess moderate preferences for enrolling their children in schools with higher minority shares $\left(\hat{\beta}^{gM} > 0\right)$. However, these estimates may be biased if, for instance, parents tend to enjoy the same school amenities as other parents of the same race.

In the second specification of Table 1, we implement our identification strategy by using $s_{jt-2}^{\bar{g}_j-1}$ as an instrument for s_{jt-1} and including C_{jt-1}^{gr} as controls. Because of the large numbers of parameters, we also present these estimates from our preferred specification in graphical form in Fig. 4. We estimate a very strong first stage (the coefficient on the instrument has a t-statistic of 95.5).[24] We still find that Whites respond negatively and minorities respond positively to high minority shares, but the responses are smaller in magnitude than the ones obtained by OLS. White preferences for peers are strongest in kindergarten and again in high school. We speculate that the former reflects the fact that parents anticipate that their kindergarten enrollment choice may have longer reaching implications in the future (due, for example, to moving costs), whereas the latter may reflect the fact that White parents are more attentive with high school choice.[25] On the other hand, minority preferences for peers are strongest in the middle school and late high school grades.[26]

5.3. Simulation results

We present graphical simulations of the expected minority share in three selected LA County schools for 2009 in Fig. 5 to highlight the direct connection between the Schelling model and our empirical method. These simulations are empirical analogs to Fig. 2. Each school exhibits qualitatively different tipping behavior; theory suggests that these differences arise because of differences in the levels of other amenities of the schools.

The simulated figure for Clifford D. Murray Elementary school reveals a strong pull towards a heavily segregated minority stable

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22 Underlying our estimation is the assumption that parental preferences for the racial composition of schools do not change over the sample period. This assumption is consistent with survey findings aggregated in Bobo et al. (2012).

23 Although the inclusion of these fixed effects barely changes the estimates, it maintains the equivalence between the estimation approach described in the main text and the discrete choice estimation approach described in the appendix.

24 All 2SLS specifications in this paper have very strong first stages. They are omitted for brevity and are available upon request.

25 Caetano (2016) finds that the marginal willingness to pay for school quality is higher in elementary school and high school than in middle school among parents in Minnesota, a predominantly White state.

26 The decrease in minority parental preferences in 9th and 10th grades is consistent with heterogeneity in Black and Hispanic parents' preferences for minority peers in these grades. We document such heterogeneity in Section 7.

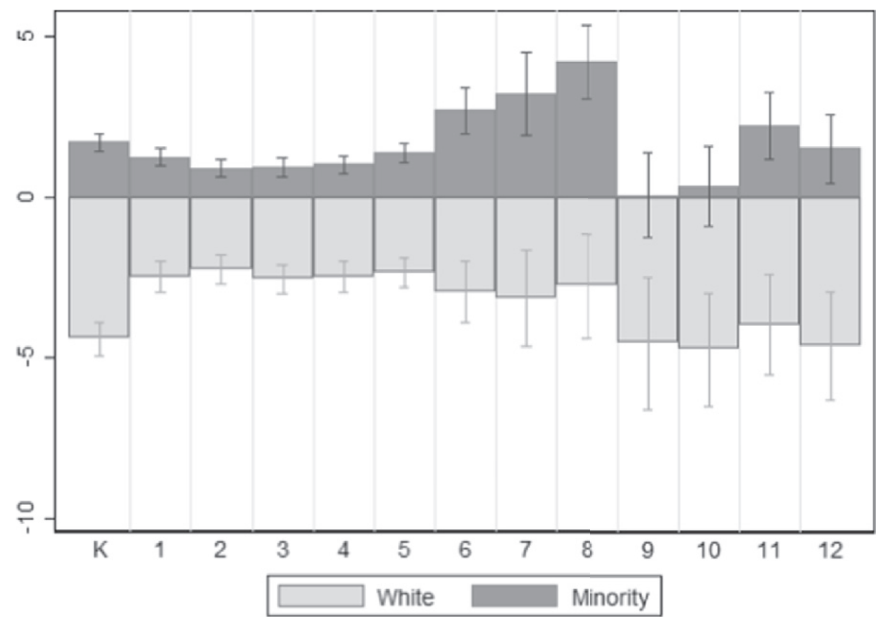

Fig. 4. 2SLS estimates of β^{gr}_{jt}. Note: Bars correspond to the estimates of β^{gr}_{jt} from the second specification of Table 1.

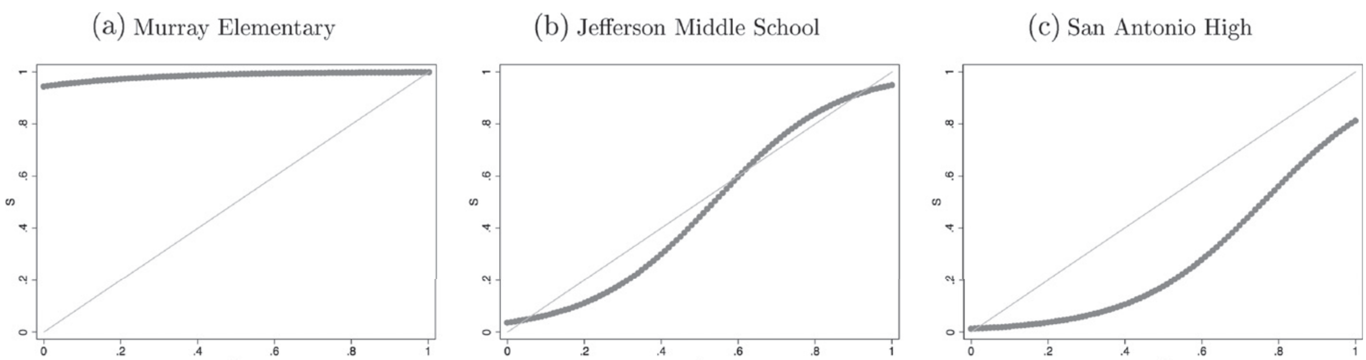

Fig. 5. Tipping diagrams: Three schools. (a) Murray Elementary. (b) Jefferson Middle School. (c) San Antonio High. Note: Each panel shows the S curve in 2009 for a specific school in the sample.

equilibrium. For any counterfactual minority share, the school will rapidly converge to an entirely minority student body. This suggests that the amenities offered by Murray Elementary are highly preferred by minority parents relative to White parents, so much so that no tipping points exist. On the other hand, Jefferson Middle School possesses an integrated tipping point near $s = 0.60$, and stable, segregated equilibria for very low and very high values of s. This figure recalls the canonical "S-curve" prescribed by the Schelling model that lends itself to multiple equilibria. Finally, San Antonio High possesses a single, heavily segregated, White stable equilibrium.

We summarize tipping behavior in the entire sample in Fig. 6. In the first panel, we consider the existence of tipping points and stable White and minority equilibria. Around 20% of schools possess tipping points, 30% of schools possess stable White equilibria, and 85% of schools possess stable minority equilibria. The prevalence of tipping is mostly unchanged over time. This stands in contrast to the empirical fact that the minority share of LA County students increased over the sample period and suggests that the racial dynamics in our sample should predominantly be understood as movements *along* schools' $S_{jt}(s)$ curves.

In the second panel of Fig. 6, we present further heterogeneity in tipping behavior with histograms of the locations of tipping points and stable equilibria.[27] Conditional on possessing a tipping point, we find that the location of tipping points varies substantially from roughly 20% minority to 80% minority. Similarly, the locations of stable equilibria also vary, though they are concentrated at highly segregated levels below 10% minority and above 90% minority.

We further explore this heterogeneity by summarizing tipping behavior in elementary schools (K–5), middle schools (6–8) and high schools (9–12) in Figs. 7–9 respectively. Tipping points are rarely observed in elementary schools, though roughly 65% of middle schools and 40% of high schools possess them. But even within school types, we find substantial heterogeneity in the locations of tipping points and stable equilibria. Although we do find heterogeneity in the locations of tipping points and stable equilibria within schools over time, it is much less pronounced than the heterogeneity in the locations of tipping points and stable equilibria between schools.

We can use our simulation results to assess the extent to which the schools in our sample are observed in equilibrium. In the first panel of Fig. 10, we present the proportion of schools that are farther than a given distance from the (relevant) stable equilibrium that

[27] Stable equilibria are doubly counted for schools that possess two of them.

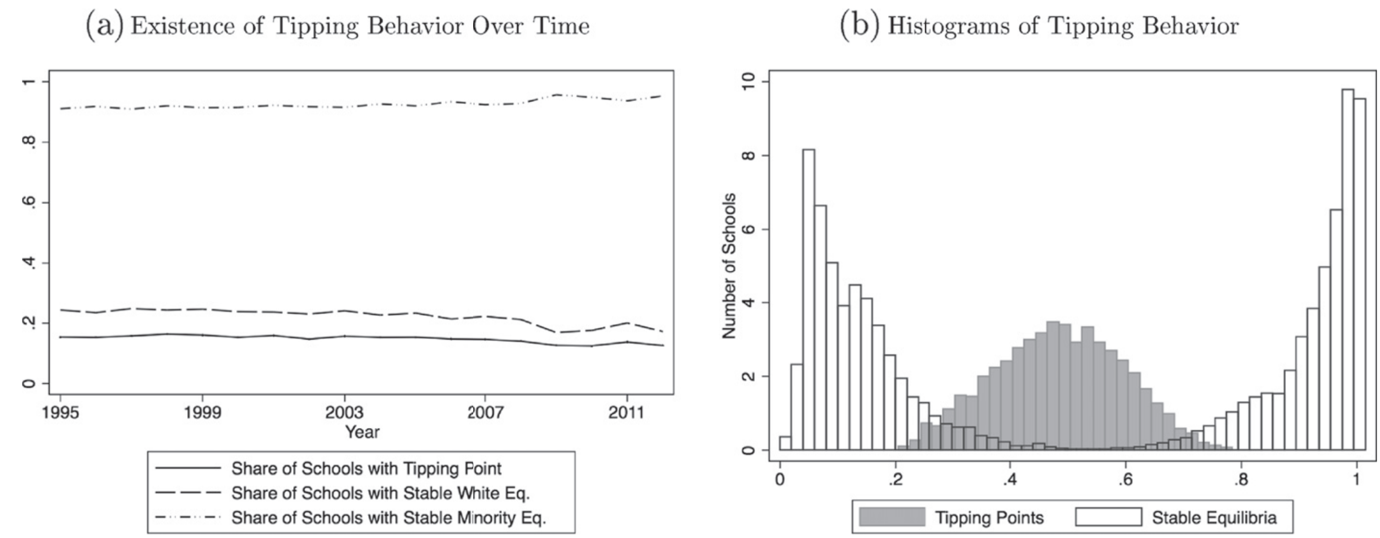

Fig. 6. Tipping behavior: All schools, 1995–2012. (a) Existence of tipping behavior over time. (b) Histograms of tipping behavior. Note: Panel (a) shows the share of schools that possess a tipping point, a stable White ($s < 0.5$) equilibrium and a stable minority ($s > 0.5$) equilibrium in each year. Panel (b) overlays histograms of tipping points and stable equilibria across all school-year combinations.

we would expect them to reach in the absence of exogenous shocks. We find that 20% of schools are observed more than 20 percentage points in minority share away from the relevant stable equilibrium, and no schools are more than 80 percentage points away. Elementary schools are more likely to be observed in equilibrium than middle schools and high schools. Over the sample period though, enrollments do tend to be observed closer to equilibrium, as depicted in the second panel of Fig. 10. Starting in 2012, we calculate that after 30 years, the racial compositions of all schools in our sample will reach within 2 percentage points of their stable equilibria in the absence of shocks.

6. Interpretation

In this section we interpret our results in the theoretical context of the Schelling model focusing on the roles of the model's primitives in explaining tipping behavior.

6.1. Tipping behavior under alternative policies

Often, policy makers are interested in the effect of an education policy on school segregation. In order to understand this effect, it is crucial to know the counterfactual level of segregation in the absence of the policy, and because schools may not be in equilibrium, this counterfactual level of segregation may even change over time in the absence of shocks. Moreover, the endogenous reaction of parents to the new racial compositions of schools (i.e., β_{jt}^{gr} in our analysis) may lead the short-run effect of a policy to be very different than its long-run effect. Our analysis can be used to better understand these effects. To illustrate this point, we present the short- and long-run effects of two different policies in Fig. 11.

The first policy that we consider is one that affects the exogenous amenities of a school (X_j from Eq. (1)). An example of such a policy would be a change in resources for programs that are preferred differently by Whites and minorities (e.g., ESL programs). A

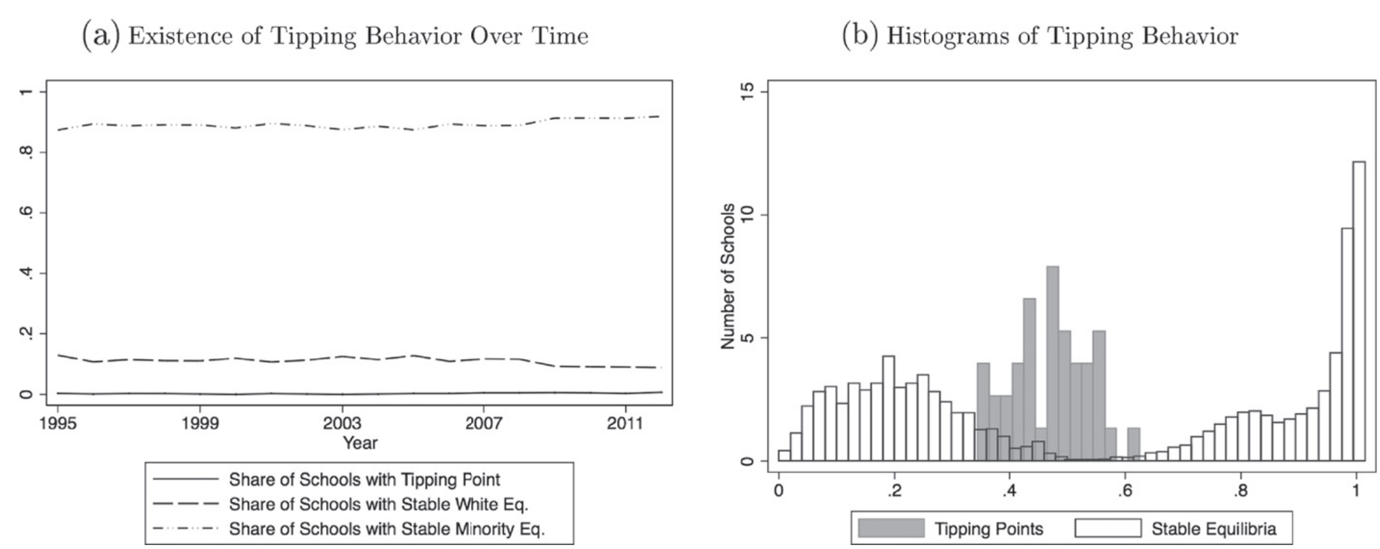

Fig. 7. Tipping behavior: Elementary schools (K–5), 1995–2012. (a) Existence of tipping behavior over time. (b) Histograms of tipping behavior. Note: Panel (a) shows the share of K–5 schools that possess a tipping point, a stable White ($s < 0.5$) equilibrium and a stable minority ($s > 0.5$) equilibrium in each year. Panel (b) overlays histograms of tipping points and stable equilibria across all K–5 school-year combinations.

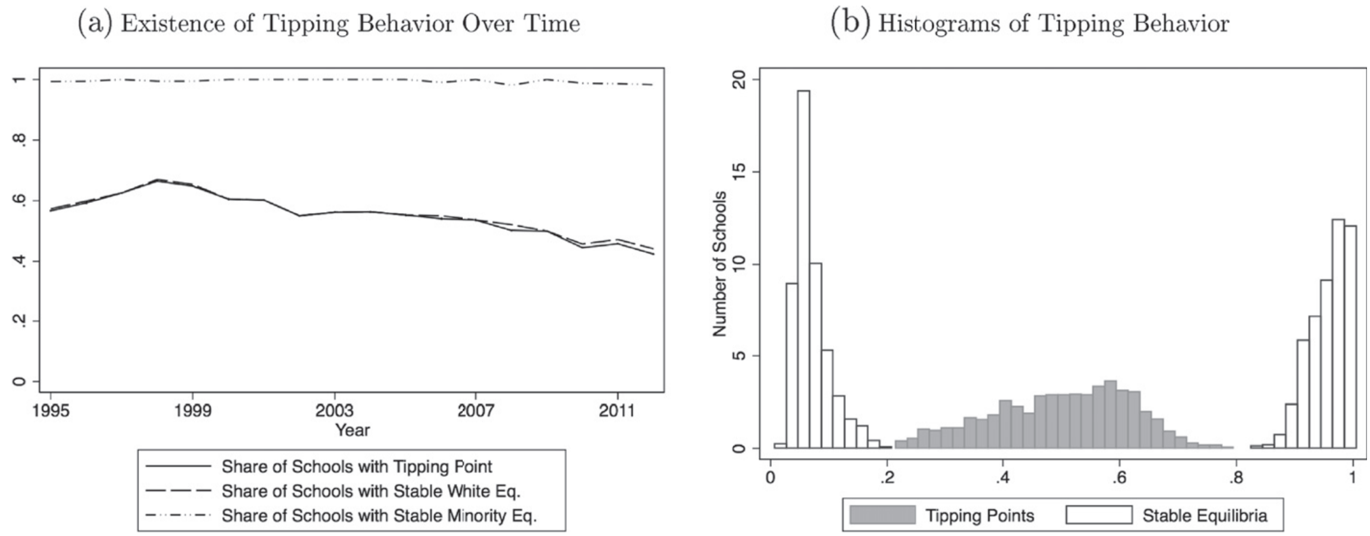

Fig. 8. Tipping behavior: Middle schools (6–8), 1995–2012. (a) Existence of tipping behavior over time. (b) Histograms of tipping behavior. Note: Panel (a) shows the share of 6–8 schools that possess a tipping point, a stable White ($s < 0.5$) equilibrium and a stable minority ($s > 0.5$) equilibrium in each year. Panel (b) overlays histograms of tipping points and stable equilibria across all 6–8 school-year combinations.

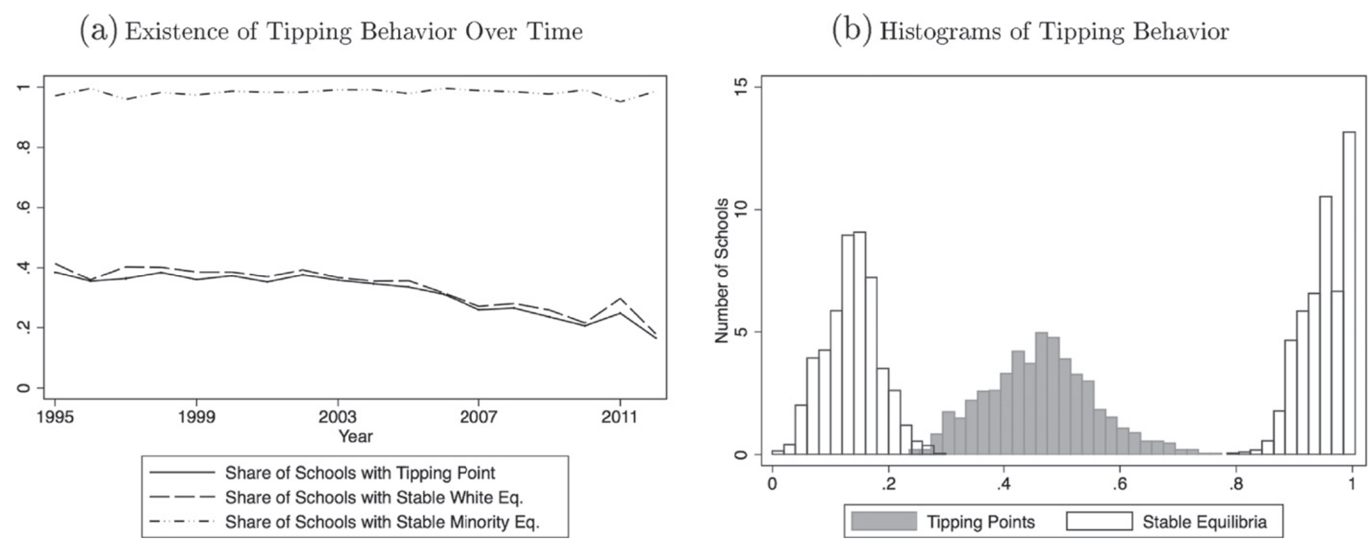

Fig. 9. Tipping behavior: High schools (9–12), 1995–2012. (a) Existence of tipping behavior over time. (b) Histograms of tipping behavior. Note: Panel (a) shows the share of 9–12 schools that possess a tipping point, a stable White ($s < 0.5$) equilibrium and a stable minority ($s > 0.5$) equilibrium in each year. Panel (b) overlays histograms of tipping points and stable equilibria across all 9–12 school-year combinations.

change in these amenities corresponds to a shift in the $S_j(\cdot)$ curve of a school, which we depict in Panel (a) as a shift to the dotted curve. If the school's minority share was originally at $s = A$, then this policy would have a short-run effect of SR (the difference between $S_j(A)$ from the new curve and the old curve). However, because the policy would change the location of the stable equilibrium, the policy would have a long-run effect of $LR < SR$.

The second policy that we consider is one that directly affects the endogenous amenity of a school, i.e., its racial composition. An example of such a policy could be the busing of White or minority students to a school. Such a policy corresponds to a movement along the $S_j(\cdot)$ curve, which we depict in Panel (b). Suppose the school's minority share was originally at $s = A$. If minority students were bused in until $s = B$, we would expect a short-run effect of $SR_B = B - A$. However, because the location of the stable equilibrium is unchanged, the endogenous responses of parents would undo this entirely leading to a long run effect of $LR_B = 0$. If instead White students were bused in until $s = C$, we would expect a short-run effect of $SR_C = A - C$ and a long-run effect of LR_C as the school would now be on a trajectory to a different stable equilibrium.[28] With this framework, policymakers may be able to anticipate these changes and design policies aimed at achieving a given specific level of segregation in a more efficient way.

[28] Note that if the policy was implemented in period t, then the "short-run" effect of policy 1 would be observed in $t + 1$, whereas the "short-run" effect of policy 2 would be observed in t.

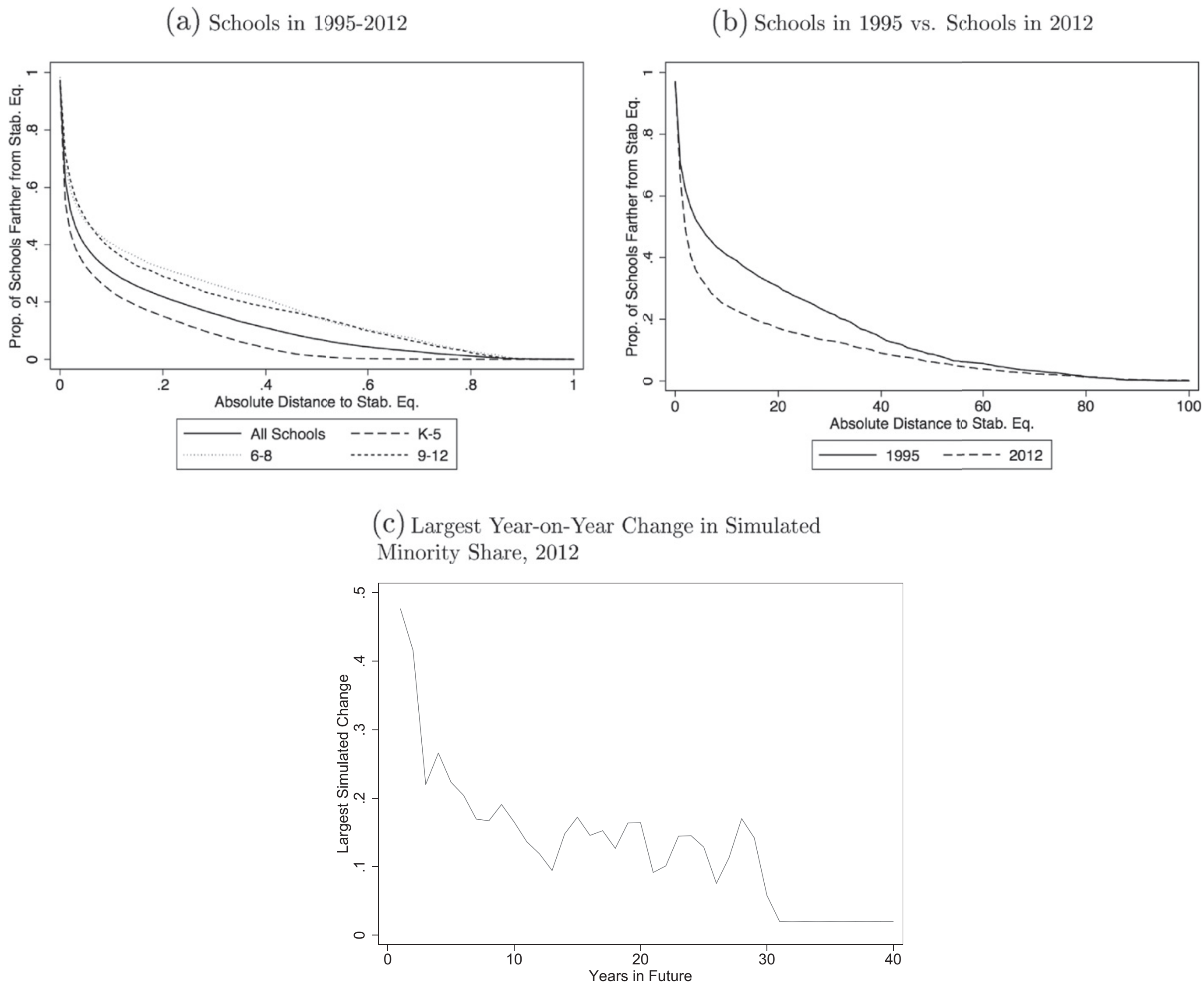

Fig. 10. Distance to stable equilibrium, 1995–2012. (a) Schools in 1995–2012. (b) Schools in 1995 vs. schools in 2012. (c) Largest year-on-year change in simulated minority share, 2012. Note: In Panel (a), for each value on the *x*-axis, we show the share of school-year combinations that are observed with a minority share that is out of equilibrium by more than *x*. For instance, about 20% of all schools and 15% of all K–5 schools are located farther than 20 percentage points out of equilibrium from the stable equilibrium to which they will eventually converge in the absence of external shocks. Panel (b) shows an inward shift of this curve from 1995 to 2012, reflecting the fact that on average, schools are approaching a stable equilibrium over the sample period. In Panel (c), we simulate the changes in the minority share of each school 40 years into the future starting from 2012 and present the largest change in minority share of any of the schools in our sample.

6.2. Tipping behavior under alternative preferences

To illustrate the role of preferences for the minority share of enrollment in explaining tipping behavior, we take Jefferson Middle School in 2009, hold all levels of (and preferences for) other amenities fixed, and simulate tipping diagrams under various counterfactual choices of β^{gr}. These exercises can also be interpreted as placebo tests for our empirical approach in order to show how the shapes of the curves would differ with different levels of endogenous social effects. Results are presented in Fig. 12.

In Panels (a) and (b), we illustrate that tipping behavior requires parents of different groups to have different preferences for social amenities. When White and minority parents value the racial composition of schools equally, schools do not feature tipping behavior. Instead, they immediately adjust to a single stable equilibrium regardless of the past racial composition.

In Panel (c), we consider tipping behavior under a counterfactual in which White parents have a preference for minority peers, and minority parents have a preference for White peers. The school now possesses a single stable equilibrium to which the racial composition will converge by oscillating. For instance, if the school began mostly White, then in the following period, it would experience relative outflows of White students and inflows of minority students. Because this new racial composition is relatively appealing to White parents, then in the period following that, the school would experience relative inflows of White students and outflows of minority students, and so on. At each period, the enrollment will move closer to the stable equilibrium. In Panel (d), we consider a similar counterfactual, except we assume that White parental preferences for minority peers and minority parental preferences for White peers are much stronger than in (c). In this case the school possesses a single unstable equilibrium around which the minority share oscillates and diverges.

In the final two panels, we consider tipping behavior under counterfactual preferences that are qualitatively similar to those that we estimated. In Panel (e), White and minority parental preferences for peers of the same race are modest. Depending on the levels of other

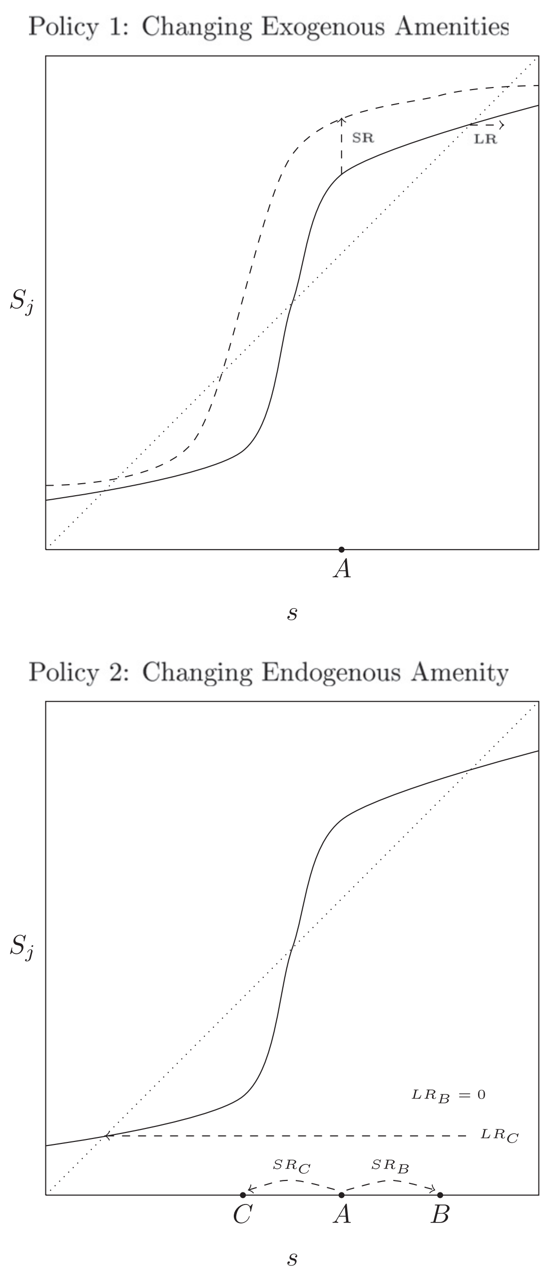

Fig. 11. Short- and long-run effects of policies. Policy 1: Changing exogenous amenities. Policy 2: Changing endogenous amenity.

amenities in the school and parental preferences for them, this could potentially generate tipping behavior. However, in Jefferson Middle School, these preferences would not have been sufficiently strong to generate tipping behavior in 2009. If, however, they were stronger, then this school would indeed feature a tipping point and two stable, segregated equilibria as in Panel (f).

6.3. Tipping behavior and other amenities

Increases in the levels of other amenities that, say, White parents enjoy relative to minority parents make schools relatively more attractive to White parents on average. This in turn causes the expected future minority share of enrollment at those schools to decrease and shifts the simulated S_{jt} curve down. The locations of tipping points and stable equilibria are affected in a predictable way. In general, a change in another amenity will shift S_{jt} even if White and minority parents have similar (though not identical) preferences for it. Hence, heterogeneity in the levels of other amenities across schools (and within schools over time), coupled with heterogeneity in preferences for those amenities implies heterogeneity in tipping behavior.[29]

In order to explore how tipping behavior is associated with differing levels of school amenities, we correlate $S_{jt}(s)$ at various values of s to three selected school amenities that are observed in the California Basic Educational Data System (CBEDS).[30] For this analysis only, we restrict the sample period from 1999 to 2012 because data on the other school amenities are not available in earlier years. We measure the academic quality of the student body using the Academic Performance Index (API)[31] and the socioeconomic status of the student body using the share of students who are eligible for a free or reduced-price lunch under the National School Lunch Program (NSLP);[32] we also consider the proportion of students with at least one parent with some college education. These three amenities proxy for characteristics of the student body that may relate to tipping behavior.

In Fig. 13, we present scatter plots of $S(0.50)$ against the levels of these three amenities. Each point corresponds to a single school-year observation. The relationship between $S(0.50)$ and these amenities reflects the direction of a shift in the S-curve in response to a change in those amenities. Schools with higher tests scores are associated with S-curves that are shifted towards the right. This suggests that White parents value school quality relatively more than minority parents. We find similar correlations with the share of students whose parents have some college education, and the opposite correlation with the share of students who are eligible for free or reduced price lunches. This suggests that White parents have a relatively stronger distaste for peers with less education and lower income than do minority parents. We summarize these correlations in Table 2, where we also report results for $S(0.25)$ and $S(0.75)$. All estimated correlations are highly statistically significant. We remark that these estimates should not be interpreted as causal; nevertheless, they are consistent with the theoretical discussion in Section 2.

7. Extension: Multiplicity of groups and social amenities

Thus far we have assumed that there are only two groups of students and that there is a single social amenity in parental demand functions. In reality, LA County contains sizable White, Black and Hispanic populations, so lumping Black and Hispanic students together may obscure interesting heterogeneity in preferences for peers and

[29] This provides a tool for policymakers to influence current and future levels of school segregation by manipulating the locations of stable equilibria and tipping points. With causal estimates of parental preferences for other amenities and the simulated curve S_{jt}, policymakers can actively adjust these amenities in school j to affect tipping behavior. By shifting the tipping point, policymakers can reverse the direction of tipping behavior in the short-run. They can also shift the relevant stable equilibrium to a more appealing location, affecting the long-run level of segregation. The identification of parental preferences for other amenities is beyond of the scope of this paper. Nevertheless, the estimation of preferences for these amenities is the objective of a large literature in public and urban economics (e.g., Chay and Greenstone, 2005) that complements our empirical framework.

[30] We choose to correlate amenities to levels of the S_{jt} curves instead of tipping points and stable equilibria because not all schools possess tipping points; hence sample selection would otherwise confound our interpretation.

[31] The base API is an accountability measure devised by the California State Board of Education that is specifically designed to compare overall performance across different schools and within schools over time.

[32] A student qualifies for a free lunch if their family's income is below 130% of the federal poverty threshold or a reduced-price lunch if their family's income ranges from 130% to 185% of the federal poverty threshold; as such, this variable is a natural proxy for the average income level of a school's student body.

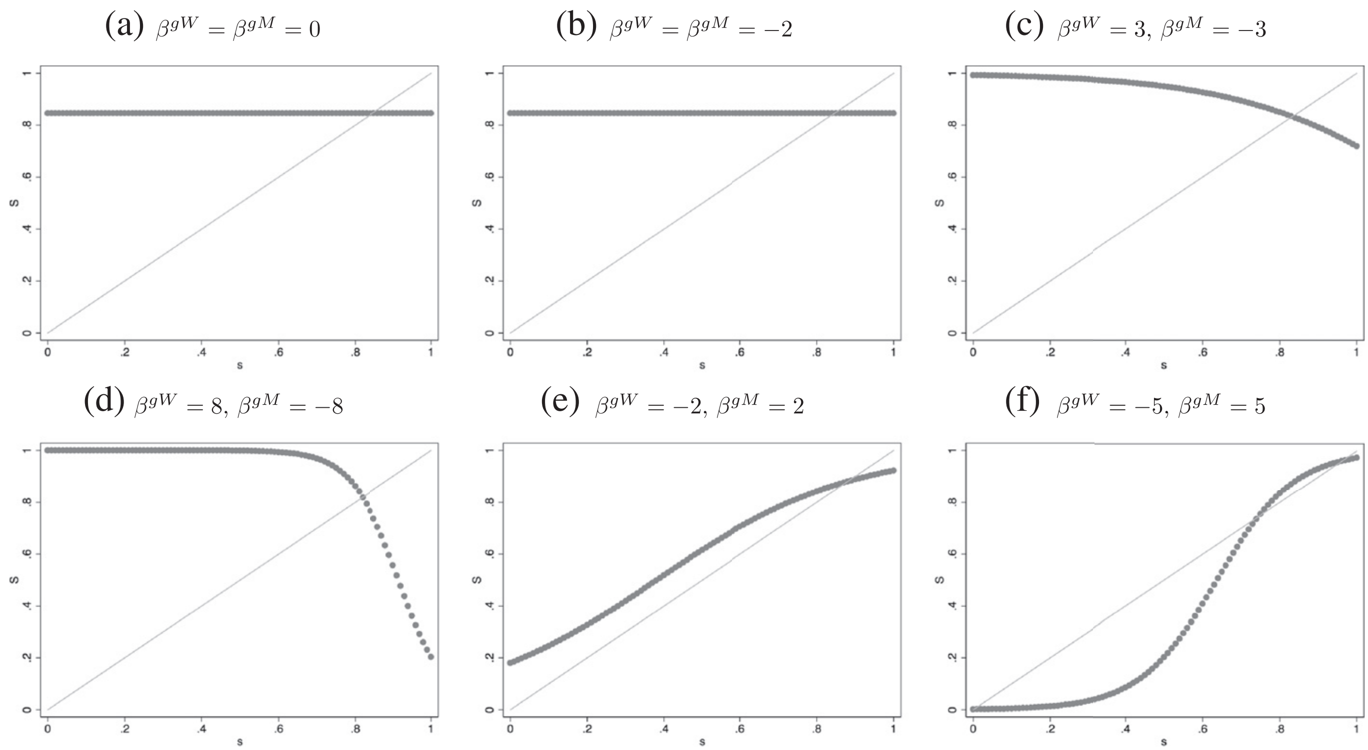

Fig. 12. Tipping figures for Jefferson Middle School under alternate estimates of β^{gr}, 2009. Note: Each panel shows the S curve under counterfactual values of β^{gW} and β^{gM}. In each counterfactual, β^{gr} is assumed to be the same for all grades. These panels can be compared to Panel (b) of Fig. 5, which reflects the S curve of the same school under the true estimates of β^{gW} and β^{gM}.

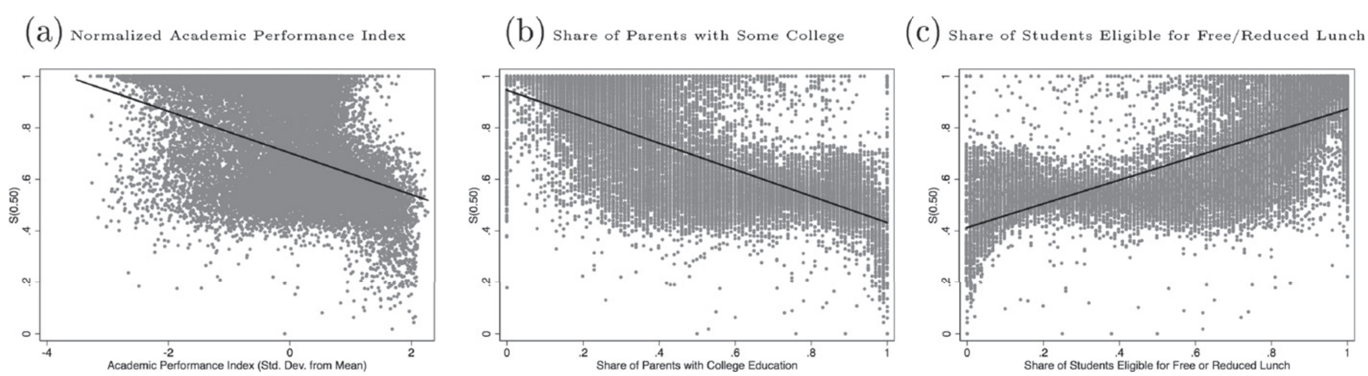

Fig. 13. Correlations of $S(0.50)$ and other amenities, 1995–2013. (a) Normalized Academic Performance Index. (b) Share of parents with some college. (c) Share of students eligible for free/reduced lunch. Note: In each plot, $S(0.50)$ (the resulting value of the $S(s)$ curve for $s = 0.50$) is on the y-axis and the level of the amenity is on the x-axis.

in tipping behavior.[33] In order to explore this possibility, we can modify demand Eq. (3) as

$$\log n_{jt}^{gr} = \beta_B^{gr} s_{jt-1}^{B} + \beta_H^{gr} s_{jt-1}^{H} + \alpha_{zt}^{gr} + f\left(n_{jt-1}^{-\bar{g}_j W}, n_{jt-1}^{-\bar{g}_j B}, n_{jt-1}^{-\bar{g}_j H}\right) + \epsilon_{jt}^{gr} \quad (8)$$

for $r \in \{W, B, H\}$ and estimate three distinct demand functions.

Specifying multiple social amenities, s_{jt-1}^{B} and s_{jt-1}^{H}, deepens the analysis in two directions. First, it allows us to implicitly test which social amenity is chiefly responsible for tipping behavior. For example, if the preference parameters for peers of one race are statistically indistinguishable from each other across groups, while the preference parameters for peers of another race are estimated to be distinct across groups, then tipping behavior (if it exists) will be due to the students of the latter race. Second, if multiple social amenities are potentially responsible for tipping behavior, then even a log-linear specification of demand may generate exotic tipping behavior.

To illuminate this second point, note that in this modified model, tipping is now a higher dimensional phenomenon. There are two implied enrollments that we must simulate, S_{jt}^{B} and S_{jt}^{H}, each of which is a function of both s^B and s^H. As a result, S_{jt}^{B} and S_{jt}^{H} are two-dimensional surfaces, and tipping points and equilibria in school j are the intersections of these two surfaces with the "45-degree" hyperplane defined by the system of equations

$$S_{jt}^{B}\left(s^B, s^H\right) = s^B \quad (9)$$

[33] In this section only we group Asian, Pacific Islander, American Indian and Alaskan Native parents with White parents, although the results are robust when we exclude these parents from the analysis. While it is computationally feasible to identify tipping behavior among more than three racial groups using our method, we do not present such results as they are difficult to show graphically.

Table 2
Correlates of S-curves, 1999–2012.

Variable	Academic Performance Index	Share of students having a parent with some college education	Share of students eligible for a free or reduced price lunch
$S(0.25)$	−0.101**	−0.684**	0.628**
	(0.004)	(0.014)	(0.011)
$S(0.50)$	−0.081**	−0.514**	0.461**
	(0.003)	(0.009)	(0.008)
$S(0.75)$	−0.051**	−0.281**	0.243**
	(0.002)	(0.007)	(0.007)

Notes: This table presents the correlation of specific points of the S_j curve with selected school amenities included in X_j. Pairwise correlations are presented along with standard errors clustered at the school level.
** Statistically significant at the 99% level.

$$S_{jt}^{H}\left(s^{B},s^{H}\right)=s^{H} \tag{10}$$

in the four dimensional (S^B, S^H, s^B, s^H) space.

We estimate the parental demand system for schooling given in Eq. (8). Instruments are constructed as before. Regression results for the preferred specification are presented in Table 3 for all specifications.[34] White parents have a distaste for both Black and Hispanic peers with a slightly stronger distaste for Black peers. Black parents show a moderate preference for Black peers but no preference for Hispanic peers except in 9th and 10th grades. On the other hand, Hispanic parents tend to show a slight preference for Hispanic peers and little if any distaste for Black peers. While these results are consistent with our baseline findings that White parents show a moderate distaste for minority peers and minority parents show a slight preference for minority peers, they reveal more complex racial preferences that we believe lend further realism to our estimation results.[35]

The heterogeneity in preferences across groups and social amenities manifests itself in richer tipping behavior. To provide a flavor of this, we present a pair of three dimensional representations of the functions S_{jt}^B and S_{jt}^H for Adams Middle School in 2009 in Fig. 14. Points where the tipping surface crosses the diagonal plane from below represent tipping points in either s^B or s^H, and points where the tipping surface crosses the diagonal plane from above represent "partially" stable equilibria in s^B or s^H (since they may not also be stable equilibria in the other social amenity). Points at which both tipping surfaces are crossed from below represent true stable equilibria.

It is difficult to glean tipping behavior from a pair of three-dimensional plots. However, we can combine both of these results in a single, digestible quiver plot that describes how the dynamic system of school enrollment changes over time. To illustrate, we present such quiver plots for the same three schools represented in Fig. 15. For any point in the triangular domain, we can simulate how the racial composition of a given school will evolve over the next period. This evolution is captured in an arrow, where longer arrows represent larger shifts in composition.

In the first panel, Murray Elementary possesses two stable equilibria. From most starting points, Murray Elementary will eventually reach a fully Hispanic student body. However, if Murray Elementary begins with a fully Black student body, the composition will not change. These two stable equilibria are both consistent with the single minority stable equilibrium that we found in the baseline simulation and presented in Fig. 5, so this constitutes a refinement of our earlier results.

[34] We subjected our specification with multiple social amenities to the same robustness checks as our preferred baseline specification and found similar results.
[35] Our estimates of White and Black parental preferences are consistent with survey evidence from Los Angeles that indicates "Black respondents are the least likely to object to residential integration" while "White respondents are the most likely to object to interracial residential contact" (Bobo and Zubrinsky, 1996).

Table 3
Parental demand for schooling, 1995–2012: Two social amenities.

	Coeff. on s_{jt-1}^B			Coeff. on s_{jt-1}^H		
r:	W	B	H	W	B	H
β^{Kr}	−4.43**	4.40**	−0.21	−4.45**	0.43	1.86**
	(0.56)	(0.42)	(0.25)	(0.28)	(0.39)	(0.18)
β^{1r}	−3.13**	1.59**	−0.59**	−2.37**	−0.19	1.17**
	(0.49)	(0.36)	(0.22)	(0.25)	(0.32)	(0.15)
β^{2r}	−3.20**	1.84**	−0.41	−2.04**	−0.17	0.99**
	(0.48)	(0.32)	(0.22)	(0.24)	(0.30)	(0.16)
β^{3r}	−2.73**	1.64**	−0.58*	−2.53**	−0.56	0.96**
	(0.48)	(0.32)	(0.25)	(0.24)	(0.31)	(0.16)
β^{4r}	−3.00**	1.89**	−0.79**	−2.22**	0.16	0.93**
	(0.49)	(0.34)	(0.28)	(0.25)	(0.30)	(0.16)
β^{5r}	−2.90**	1.48**	−1.15**	−2.15**	−0.05	1.43**
	(0.48)	(0.35)	(0.29)	(0.24)	(0.29)	(0.16)
β^{6r}	−6.67**	3.28**	−1.43**	−2.74**	0.32	2.53**
	(0.98)	(0.87)	(0.75)	(0.49)	(0.56)	(0.38)
β^{7r}	−6.78**	2.54**	−1.50	−2.56**	0.63	3.39**
	(1.39)	(1.09)	(1.11)	(0.79)	(0.78)	(0.80)
β^{8r}	−5.33**	3.16**	0.51	−2.28**	0.90	5.10**
	(1.30)	(1.15)	(1.05)	(0.83)	(0.83)	(0.67)
β^{9r}	−3.35*	1.88	−2.91*	−4.46**	−4.88**	−0.02
	(1.69)	(1.80)	(1.22)	(1.05)	(1.20)	(0.74)
β^{10r}	−5.25**	1.53	−1.22	−4.31**	−2.66**	0.60
	(1.68)	(1.54)	(1.07)	(0.89)	(1.05)	(0.74)
β^{11r}	−6.54**	2.99*	−1.00	−3.77**	−0.08	2.98**
	(1.51)	(1.51)	(0.97)	(0.79)	(0.96)	(0.56)
β^{12r}	−6.43**	−0.38	−3.54**	−4.31**	−0.35	1.90**
	(1.56)	(1.67)	(1.20)	(0.85)	(0.99)	(0.64)
R^2			0.43			
Num. Obs.			542,799			

Notes: The dependent variable is log enrollment by grade, race, school and year $\left(\log n_{jt}^{gr}\right)$. Regression includes grade-race-year-ZIP code fixed effects and $\log n_{jt-1}^{\underline{g}_jW},\ldots,\log n_{jt-1}^{\bar{g}_j-1W}$, $\log n_{jt-1}^{\underline{g}_jB},\ldots,\log n_{jt-1}^{\bar{g}_j-1B}$ and $\log n_{jt-1}^{\underline{g}_jH},\ldots,\log n_{jt-1}^{\bar{g}_j-1H}$ as controls. $s_{t-2}^{B\bar{g}_j-1}$ and $s_{t-2}^{H\bar{g}_j-1}$ are used as instrumental variables. Robust standard errors clustered by grade, race, year and ZIP code are provided in parentheses.
* Statistically significant at the 95% level.
** Statistically significant at the 99% level.

In the second panel, Jefferson Middle School possesses an unstable equilibrium at roughly 60% Black and 2% Hispanic enrollments. This is consistent with the tipping point we found in the baseline simulation. We find stable minority equilibria at 20% Black/75% Hispanic enrollments and at 90% Black/5% Hispanic enrollments. We also find a single stable White equilibrium at 5% Black enrollment. All three of these stable equilibria are consistent with the two stable equilibria that we found in the baseline simulation.

In the third panel, San Antonio High School possess a stable equilibrium in which all students are White, which is consistent with what we found in the baseline simulation. However, it also possesses a stable all-Black equilibrium along with an unstable equilibrium at 50% White and 50% Hispanic enrollments. The latter two equilibria reflect more complex dynamics that cannot be uncovered in our baseline specification.

In general, the higher-dimensional analogs of tipping points and stable equilibria points are one dimensional manifolds, i.e. equilibria may exist along curves, so we cannot describe the distributions of tipping points and stable equilibria with unidimensional histograms. Furthermore, the computational complexity associated with identifying the full set of stable and unstable equilibria for all schools even with just two social amenities is quite considerable. For this reason, we can only present higher dimensional tipping diagrams for selected school-year combinations in our sample as a showcase of the generality of our empirical approach. Extending this analysis for the entire sample in order to summarize higher dimensional tipping behavior lies beyond the scope of this paper.

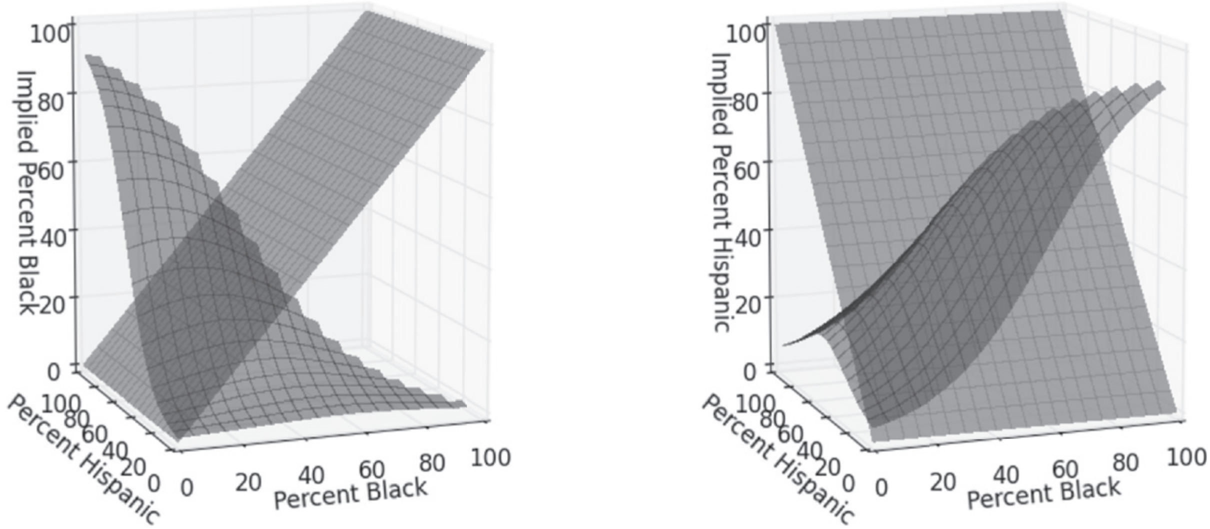

Fig. 14. Jefferson Middle School, two social amenities. Note: These panels are the higher dimensional analogs of Panel (b) of Fig. 5, which present the S" curve" of Jefferson Middle School under the assumption that Black and Hispanic students are indistinguishable.

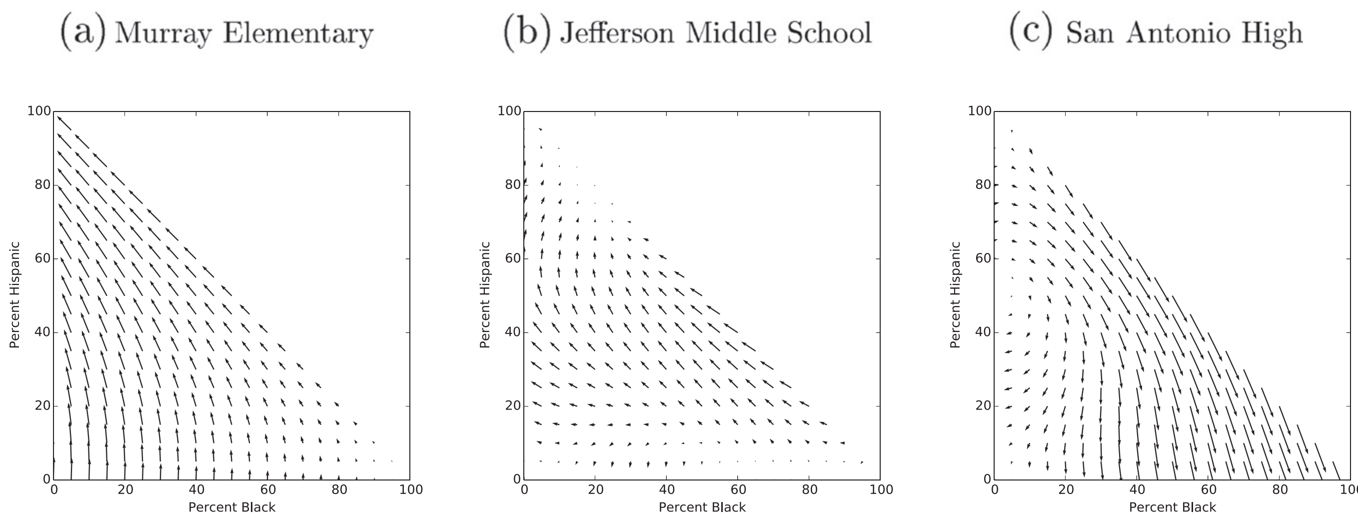

Fig. 15. Quiver plots: Two social amenities. (a) Murray Elementary. (b) Jefferson Middle School. (c) San Antonio High. Notes: All results from 2009. For all points in the domain, we simulate how the racial composition of this school will evolve to next year. Longer arrows represent larger shifts in the racial composition. These panels can be compared to the panels of Fig. 5, which make the restrictive assumption that Black and Hispanic students are indistinguishable.

8. Conclusion

The Schelling model of segregation explains seemingly complex dynamics in a simple framework. In this paper, we provide an empirical implementation of this model with novel features. Our approach to identify tipping points and stable equilibria allows for heterogeneity in their existence and their location and can be adapted to analyze tipping behavior in more complex settings. In addition, we introduce an IV approach to identify preferences for peer groups in educational settings with two valuable features. First, it requires relatively sparse data and can be implemented with just a panel of grade level school enrollments. Second, the logic of our approach can be leveraged to directly test our identifying assumptions with falsification and over-identification tests. This IV approach can be of use in a growing number of studies of peer effects in educational settings.

We implement our approach in the case of racial segregation in public schools and find that the market for public schooling in Los Angeles County from 1995 to 2012 was both diverse and dynamic. Parents have strong preferences for peers of the same race, particularly in later grades. Thus, we tend to find tipping behavior in middle and high schools but not in elementary schools. We document substantial heterogeneity in the tipping behavior across schools that is correlated with the school's quality and the school's socioeconomic composition. We also find that many schools are observed far from a stable equilibrium, although they seem to be in the process of convergence to it. Finally, the structure of our empirical approach allows us to analyze tipping in a higher-dimensional context that captures realistic heterogeneity in the preferences of Black and Hispanic parents.

Our paper highlights the need for a more conservative view of the state of the art in the subject of tipping behavior and segregation, as better data are needed to undertake a more complete analysis on the topic. With improved data, it would be useful to explore further substitution effects between schools along the lines of Berry et al. (1995) and Bayer and McMillan (2010) and to allow for the separate identification of moving costs and preferences as in Bayer et al. (2016). In addition, the availability of price data would allow us to explore the complex interactions that have been documented between peer groups and prices (Kiel and Zabel, 1996). We believe that the approach developed here serves as a platform upon which these features can be added.[36]

Appendix A. Discrete choice demand estimation

In this section we provide micro-foundations to our demand estimation in line with the literature on school and neighborhood discrete choice models (e.g., Bayer and Timmins, 2005; Bayer et al., 2007; Caetano, 2016).

A.1. Basic setup

In each year t, parents make a choice of whether to enroll their child in one of J available public schools in Los Angeles County or instead to select the outside option of enrolling their child in a private school or living outside of LA County altogether. We assume that school supply is perfectly elastic.[37] In addition, for simplicity in this appendix we assume that there are no moving costs associated with parent i's enrollment decision, so it suffices to consider the flow utility of making each choice.[38]

Parents make their enrollment decisions in period t having observed the set of school amenities at the end of period $t-1$. This may include the amenities of the neighborhood where parents would need to live in order to enroll their child in a particular school. We specify the indirect utility of parent i whose child of race r is enrolled in grade g at school j in year t as

$$U_{ijt}^{gr} = \beta^{gr} s_{jt-1} + \xi_{jt}^{gr} + \eta_{ijt}^{gr} \tag{A.1}$$

where all variables and parameters are as described in the main text. ξ_{jt}^{gr} represents an unobservable that varies at the grade-race-school-year level, and the error term η_{ijt}^{gr} is an individual specific unobserved component of utility that is assumed to be i.i.d. extreme value 1.[39] Each individual chooses among $J+1$ options, where $j=0$ indexes the outside option (interpreted as the choice to send the child to a private school or a public school outside of LA County), and $j=1,\ldots,J$ indexes each public school in LA County.

Parent i chooses school j in year t if

$$U_{ijt}^{gr} > U_{ikt}^{gr}, \tag{A.2}$$

for all alternatives $k \neq j$ including the outside option.

A.2. First stage: Estimation

We first collect the non-individual specific determinants of utility into $\delta_{jt}^{gr} \equiv \delta_{jt}^{gr}(s_{jt-1}) = \beta^{gr} s_{jt-1} + \xi_{jt}^{gr}$. We normalize $\delta_{0t}^{gr} = 0$ for each g, r and t. It follows from Eq. (A.2) that parent i will enroll their child in school j at t if $\eta_{ikt}^{gr} - \eta_{ijt}^{gr} < \delta_{jt}^{gr} - \delta_{kt}^{gr}$ for all $k \neq j$ including the outside option. We denote this probability of enrollment as P_{ijt}^{gr}. The assumption on the distribution of η implies that P_{ijt}^{gr} is constant within grade, race, school and year, so we can drop the subscript i and write this probability as

$$P_{jt}^{gr}(s_{jt-1}) = \frac{\exp\left(\delta_{jt}^{gr}(s_{jt-1})\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_{kt}^{gr}(s_{kt-1})\right)}, \quad j = 1,\ldots,J \tag{A.3}$$

which is the familiar logit relationship. Following Berry (1994), we can estimate each δ_{jt}^{gr} as

$$\hat{\delta}_{jt}^{gr} = \log \frac{n_{jt}^{gr}}{n_{0t}^{gr}} \tag{A.4}$$

directly from the data. $\hat{\delta}_{jt}^{gr}$ can be interpreted as the estimated mean utility that race r parents enjoy from enrolling their children in grade g in school j in year t. We can then write

$$\log n_{jt}^{gr} = \beta^{gr} s_{jt-1} + \underbrace{\xi_{jt}^{gr} + \log n_{0t}^{gr} + \mu_{jt}^{gr}}_{\text{unobserved}}, \quad j = 1,\ldots,J \tag{A.5}$$

where $\mu_{jt}^{gr} = \hat{\delta}_{jt}^{gr} - \delta_{jt}^{gr}$ is the error due to estimation of δ. We can re-write Eq. (A.5) as

$$\log n_{jt}^{gr} = \beta^{gr} s_{jt-1} + \alpha_{zt}^{gr} + \epsilon_{jt}^{gr}, \quad j = 1,\ldots,J \tag{A.6}$$

where α_{zt}^{gr} is a grade-race-neighborhood-year fixed effect that allows for schools in the same neighborhood to have a different level of substitutability than schools in different neighborhoods (e.g., because of the availability of private schools in the neighborhood), and the composite error term $\epsilon_{jt}^{gr} = \xi_{jt}^{gr} + \log n_{0t}^{gr} + \mu_{jt}^{gr} - \alpha_{zt}^{gr}$. Note that α_{zt}^{gr} absorbs $\log n_{0t}^{gr}$, which crucially eliminates the need to observe n_{0t}^{gr} (which we do not observe). It follows that ϵ_{jt}^{gr} contains only grade-race-school-year specific components, which we argue to be orthogonal to the proposed instrument described in the main text. Thus, the demand estimation procedure outlined in the main text is equivalent to a standard, well defined school decision problem that embeds a neighborhood choice model, and the proposed instrument can be used to identify β^{gr}.

A.3. Second stage: Simulation

It is also the case that the simulation procedure in this discrete choice framework is equivalent to the simulation procedure described in the main text. Without loss of generality, consider the following decomposition

$$P_{ijt}^{gr} = \pi_{ijt}^{gr} \cdot P_{it}^{gr}, \quad j = 1,\ldots,J \tag{A.7}$$

where P_{it}^{gr} is the probability that parent i will not choose the outside option, and π_{ijt}^{gr} is the probability of that parent choosing school j conditional on not choosing the outside option. Given our assumptions, we can drop the index i for simplicity and denote this conditional probability π_{jt}^{gr}. Eqs. (A.3) and (A.7) then imply

$$\pi_{jt}^{gr}(s_{jt-1}) = \frac{\exp\left(\delta_{jt}^{gr}(s_{jt-1})\right)}{\sum_{k=1}^{J} \exp\left(\delta_{kt}^{gr}(s_{kt-1})\right)}, \quad j = 1,\ldots,J. \tag{A.8}$$

[36] We discuss some of these challenges in the supplementary appendix.

[37] By modeling the supply side as commonly done in discrete choice demand estimation, one could relax this assumption (see the supplementary appendix for a detailed description of this point).

[38] We do not make this assumption in the paper, since β^{gr} reflects both preferences and moving costs. With detailed data related to the transition of students between schools, one could relax this assumption and estimate a dynamic discrete choice model (Bayer et al., 2016; Caetano, 2016) and separately identify preferences and moving costs.

[39] The distribution of η_{ijt}^{gr} can be generalized following Berry et al. (1995) to account for other types of heterogeneity in preferences. However, we believe that may not be a good idea in our context even with better data (see the supplementary appendix for a detailed discussion of this topic).

For each counterfactual value s, the implied share of minority students in school j at time t, $S_{jt}(s)$, can then be written as

$$S_{jt}(s) = \frac{\sum_g N_t^{gM} \pi_{jt}^{gM}(s)}{\sum_g N_t^{gM} \pi_{jt}^{gM}(s) + N_t^{gW} \pi_{jt}^{gW}(s)}, \quad j = 1, \ldots, J, \tag{A.9}$$

where $N_t^{gr} = \sum_{k=1}^{J} n_{kt}^{gr}$, under the assumption that $\log n_{0t}^{gr}(s) = \log n_{0t}^{gr}$, so that $P_{it}^{gr}(s) = P_{it}^{gr}$.[40] Substituting Eq. (A.4) into Eq. (A.8) yields

$$\begin{aligned} \pi_{jt}^{gr}(s) &= \frac{\exp\left(\log n_{jt}^{gr}(s) - \log n_{0t}^{gr}\right)}{\sum_{k=1}^{J} \exp\left(\log n_{kt}^{gr}(s) - \log n_{0t}^{gr}\right)} \\ &= \frac{n_{jt}^{gr}(s)}{\sum_{k=1}^{J} n_{kt}^{gr}(s)}, \quad j = 1, \ldots, J. \end{aligned} \tag{A.10}$$

Finally, by substituting Eq. (A.10) into Eq. (A.9), we arrive at Eq. (7) in the main text. By holding α_{zt}^{gr} fixed in the simulation, we explicitly hold the attractiveness of each neighborhood fixed. Although our simulation procedure fixes the number of students planning to enroll in LA County public schools, it does not constrain parental substitution across neighborhoods. This provides a stationary environment suitable for the identification of fixed points.

Appendix B. Robustness

B.1. Estimation

In Table B.4, we present the results for several robustness checks of our preferred specification (2) from Table 1. The first two specifications assess the ability of our instrumental variables to isolate the endogenous social effect of interest. The following two specifications explore the extent to which our estimates should be interpreted as preferences for the racial composition of schools versus preferences for the racial composition of both schools and neighborhoods.

In specification (1), we explore the logic of our identification strategy more deeply by formally testing the validity of $s_{t-1}^{\bar{g}_j}$. Recall that $s_{t-1}^{\bar{g}_j}$ may not be a valid IV because we cannot include grade $\bar{g}_j$ enrollments as controls in C_{jt-1}^{gr} (hence permanent grade $\bar{g}_j$ specific amenities could violate our identifying assumption). We test whether this is the case by re-estimating parental demands using both $s_{t-1}^{\bar{g}_j}$ and $s_{t-2}^{\bar{g}_j-1}$ as instrumental variables for s_{jt-1}. The Hansen (1982) J-statistic from an over-identification test of the validity of these instruments has a p-value of 0.0001, which allows us to reject the exclusion restriction at a very high level of confidence. In specification (2), we conduct a similar exercise to test the validity of our preferred instrument $s_{jt-2}^{\bar{g}_j-1}$, this time by re-estimating parental demands using $s_{jt-2}^{\bar{g}_j-1}$ and $s_{jt-3}^{\bar{g}_j-2}$ as instrumental variables for s_{jt-1}. Because we include grade $\bar{g}_j-1$ and $\bar{g}_j-2$ enrollments in $t-1$ as controls, permanent amenities specific to those grades will be plausibly absorbed. Indeed, our parameter estimates are statistically indistinguishable from those in our preferred specification, and the J-statistic from an over-identification test of these instruments has a p-value of 0.58, which does not allow us to reject the exclusion restriction even at a very low confidence level.[41] Taken all together, we interpret specifications (1) and (2) as strong evidence in favor of the logic of our identification strategy and our exclusion restriction[42].

School choice and neighborhood choice are related decisions, so it is important to consider whether our estimates capture preferences for the racial composition of schools as opposed to residual preferences for the racial composition of neighborhoods. In order to explore this distinction, we re-estimate our preferred specification and include fixed effects at the grade-race-year-Census tract level. Given the smaller geographic size of Census tracts in comparison to ZIP codes, this allows us to improve our identification strategy by effectively comparing schools offering instruction in at least one common grade that are located much closer to each other. We present the results in specification (3) and obtain coefficient estimates that are similar to those estimated in our preferred specification.

Differences between these coefficient estimates and our preferred ones, however small, could arise for two reasons. First, the coefficient estimates in our preferred specification may include parental preferences for neighborhoods. Second, the inclusion of grade-race-year-Census tract fixed effects reduces the size of the sample available for estimation by roughly 80% down to 87,840 observations. Hence the small difference in coefficient estimates could simply be due to the fact that the subsample of schools that share a grade with another school in the same Census tract is not representative of all LA County schools, as they are likely located in higher density neighborhoods.

We attempt to disentangle these two reasons by re-estimating our preferred specification on the subsample of 87,840 observations, and we present the results in specification (4). These coefficient estimates are statistically indistinguishable from those in specification (3); hence we cannot reject the second explanation. We interpret this as suggestive evidence that the coefficient estimates in our preferred specification predominantly capture parental preferences for the racial composition of schools *per se*. This is unsurprising, since C_{jt-1}^{gr} likely captures many unobserved neighborhood amenities, so only neighborhood attributes that are specific to grade $\bar{g}_j$ would not be controlled for. However, a full distinction between school and neighborhood tipping is beyond the scope of this paper, as it would require student and non-student data at the school attendance area level, which is frequently smaller than a Census Tract.

In Table B.5, we present the results from a variety of additional robustness checks. In the description of our sample, we drew attention to the fact that there is substantial variation in the range of grades offered in LA County schools that is both cross-sectional and longitudinal in nature. In the first two specifications, we show that our results do not crucially rely on either of these sources of variation. First, we absorb the cross-sectional variation in the range of

[40] This assumption holds if parental substitution patterns into the outside option do not vary within neighborhood. In any case, the counterfactual minority share of enrollment in a particular school is not likely to affect the overall county level substitution into the outside option as a practical matter since the number of schools in LA County is large.

[41] There is a reasonable concern of whether over-identification tests are powerful enough to detect invalid IVs, as we might fail to reject the null when the IVs are invalid in similar ways. We offer two pieces of evidence to suggest that in our context we have a powerful test. First, we also implement all other combinations of over-identification tests with the available candidate IVs $\left(\text{e.g., } s_{jt-1}^{\bar{g}_j} \text{ and } s_{jt-3}^{\bar{g}_j-2}\right)$, and the test consistently rejects the null hypothesis when (and only when) $s_{jt-1}^{\bar{g}_j}$ is included as IV. Second, when we restrict our sample to K–12 schools only, we can likely perform a more powerful test because we can maximize the distance between the IVs in the test. When we include s_{jt-13}^{K} and s_{jt-1}^{12} as IVs, we reject the test. In contrast, we fail to reject the test when we include s_{jt-13}^{K} and s_{jt-2}^{11} as IVs.

[42] We find two additional empirical results in favor of the claim that C_{jt-1}^{gr} fully absorbs the confounding component of the IV. First, we find similar results when we control for the enrollments of each race for all grades except the highest grade more flexibly (i.e., not only as a linear combination of log-demand for each grade and race). Second, we find similar results when we also add as controls the enrollments of the control cohorts observed at earlier years $\left(C_{jt-2}^{gr}\right)$.

Table B.4
Parental demand for schooling, 1995–2012: Robustness 1.

	(1)		(2)		(3)		(4)	
r:	W	M	W	M	W	M	W	M
β^{Kr}	−4.42**	1.71**	−4.06**	1.49**	−4.22**	1.98**	−4.57**	2.49**
	(0.25)	(0.14)	(0.26)	(0.14)	(0.87)	(0.49)	(0.62)	(0.41)
β^{1r}	−2.60**	1.16**	−2.36**	1.11**	−1.29	1.05*	−1.90**	1.33**
	(0.23)	(0.13)	(0.24)	(0.13)	(0.69)	(0.49)	(0.49)	(0.36)
β^{2r}	−2.16**	0.83**	−2.20**	0.78**	−1.98**	1.15*	−1.84**	1.39**
	(0.21)	(0.13)	(0.23)	(0.13)	(0.75)	(0.49)	(0.52)	(0.37)
β^{3r}	−2.57**	0.87**	−2.55**	0.92**	−2.17**	0.44	−2.55**	1.36**
	(0.22)	(0.14)	(0.23)	(0.15)	(0.68)	(0.48)	(0.53)	(0.37)
β^{4r}	−2.47**	0.91**	−2.61**	0.83**	−1.54*	0.88	−1.99**	1.26**
	(0.22)	(0.12)	(0.24)	(0.13)	(0.76)	(0.47)	(0.54)	(0.34)
β^{5r}	−2.34**	1.29**	−2.62**	1.20**	−1.20	2.20**	−1.41	2.24**
	(0.22)	(0.14)	(0.23)	(0.14)	(0.75)	(0.52)	(0.54)	(0.41)
β^{6r}	−2.17**	3.10**	−1.70**	2.17**	−3.80**	1.95	−2.96**	2.66**
	(0.44)	(0.32)	(0.54)	(0.43)	(1.23)	(1.15)	(0.90)	(0.65)
β^{7r}	−4.68**	3.02**	−5.72**	2.29*	−4.58	5.82*	−6.16**	4.41*
	(0.94)	(0.57)	(1.45)	(1.04)	(3.05)	(2.51)	(2.36)	(1.97)
β^{8r}	−4.34**	4.33**	−4.16**	5.13**	−6.48*	5.52**	−6.75**	4.41**
	(0.83)	(0.53)	(1.48)	(1.16)	(3.13)	(1.73)	(2.59)	(1.32)
β^{9r}	−4.68**	−0.67	−5.98**	−0.57	−11.6**	−1.37	−9.38**	−1.21
	(0.94)	(0.58)	(1.11)	(0.70)	(2.44)	(1.43)	(1.89)	(1.04)
β^{10r}	−4.34**	0.08	−3.38**	0.87	−8.06**	0.84	−6.65**	0.42
	(0.83)	(0.52)	(0.87)	(0.52)	(1.80)	(1.36)	(1.44)	(1.02)
β^{11r}	−3.06**	1.59**	−4.07**	1.71**	−4.56**	3.52**	−5.57**	2.47**
	(0.72)	(0.44)	(0.85)	(0.55)	(1.48)	(0.98)	(1.35)	(0.80)
β^{12r}	−4.20**	1.40**	−5.65**	1.25*	−7.05**	2.14*	−8.30**	1.37
	(0.75)	(0.47)	(0.87)	(0.61)	(1.88)	(1.09)	(1.50)	(0.88)
Controls?	Yes		Yes		Yes		Yes	
IVs?	$S_{t-1}^{\bar{g}_j}, S_{t-2}^{\bar{g}_j-1}$		$S_{t-2}^{\bar{g}_j-1}, S_{t-3}^{\bar{g}_j-2}$		$S_{t-2}^{\bar{g}_j-1}$		$S_{t-2}^{\bar{g}_j-1}$	
Gr.-race-year-ZIP code FEs?	Yes		Yes		No		Yes	
Gr.-race-year-census tract FEs?	No		No		Yes		No	
J-statistic	61.66**		24.00		–		–	
(p-Value)	(0.00)		(0.58)					
R^2	0.43		0.42		0.46		0.45	
Num. obs.	361,866		314,400		87,840		87,840	

Notes: The dependent variable is log enrollment by grade, race, school and year $\left(\log n_{jt}^{gr}\right)$. All specifications are estimated by 2SLS and include $\log n_{jt-1}^{\underline{g}_j W}, \ldots, \log n_{jt-1}^{\bar{g}_j-1W}$ and $\log n_{jt-1}^{\underline{g}_j M}, \ldots, \log n_{jt-1}^{\bar{g}_j-1M}$ as controls. J-statistics in (1) and (2) are reported from Hansen's over-identification test. Coefficients in (4) are estimated on a restricted subsample that is identical to (3). Robust standard errors clustered by grade, race, year and ZIP code are provided in parentheses.

* Statistically significant at the 95% level.

** Statistically significant at the 99% level.

grades by re-estimating our preferred specification with fixed effects at the grade-race-year-ZIP code-grade range level and present the results in specification (1).[43] Second, we eliminate the longitudinal variation in the range of grades by re-estimating our preferred specification on a restricted subsample of school-years that have no change in the grades offered from $t-1$ to t. In both specifications we obtain coefficient estimates that are broadly similar to those of our preferred specification. Moreover, the standard errors are only slightly larger than in our preferred specification. These results suggest that the observed variation in the range of grades offered, although useful, does not drive our results.

We also note that some schools offer instruction to only a very small number of students in a particular grade; such schools may not be comparable across other unobserved dimensions to other schools that offer instruction to many students in these grades. In addition, such low enrollment numbers could be subject to measurement error. As such, we re-estimate our preferred specification on restricted subsamples that exclude all observations for which fewer than 10 and 30 students are enrolled in a given grade at a given school in a given year and present the results in specifications (3) and (4), respectively. Again, the coefficient estimates are broadly similar and statistically indistinguishable from those in our preferred specification.[44]

[43] One potential concern is that students of the IV cohort may have younger siblings who enter the school in t, thereby confounding our estimates. (If these siblings enter the school before t, their enrollment is absorbed by our controls.) Grade-race-year-ZIP code-grade range fixed effects absorb any reminding confounding effect related to them. To see this, suppose we compare estimates of, say, β^{Kr} from regressions on two different subsamples: one that includes only K–5 schools and the other that includes only K–12 schools. Because the IV cohorts in these two subsamples are separated by 7 years of age, the students of the IV cohort in K–5 schools are 6 years older than their siblings entering kindergarten, while the students of the IV cohort in K–12 schools are 13 years older than their siblings entering kindergarten. The magnitude of this potentially confounding effect should vary with the relative frequency of families with each amount of birth spacing, so any ensuing bias should vary with the grade range of the schools in our sample.

[44] We also addressed two additional potential concerns related to confounding cohort effects. The first potential concern is that cohort effects of those students in the lowest grade of a school ($\underline{g}_j$) in t are not explicitly controlled for by C_{jt-1}^{gr}. Because of the variation in $\underline{g}_j$ in our sample, the estimates $\hat{\beta}^{gr}$ for all grades g except kindergarten are all estimated off observations from a mix of school-grades with $\underline{g}_j = g$ and $\underline{g}_j < g$. Since this issue would only bias our results because of school-grades with $\underline{g}_j = g$, we compared our main estimates with estimates from the restricted sample of school-grades with $\underline{g}_j < g$ and found no statistically significant differences. The second potential concern is that students in the IV cohort of different races might repeat the highest grade of the school at different rates. Since this issue would only bias our results because of school-grades with $\bar{g}_j = g$, we followed the same logic as above and compared our main estimates with estimates from the restricted sample of school-grades with $\bar{g}_j > g$. Again, we found no statistically significant differences.

Table B.5
Parental demand for schooling, 1995–2012: Robustness 2.

	(1)		(2)		(3)		(4)	
r:	W	M	W	M	W	M	W	M
β^{Kr}	−3.98**	1.28**	−4.61**	1.29**	−4.21**	1.63**	−4.30**	1.52**
	(0.29)	(0.18)	(0.27)	(0.14)	(0.26)	(0.11)	(0.26)	(0.10)
β^{1r}	−1.94**	1.32**	−2.77**	0.98**	−2.22**	1.45**	−2.35**	1.43**
	(0.29)	(0.16)	(0.22)	(0.11)	(0.24)	(0.10)	(0.24)	(0.10)
β^{2r}	−1.86**	0.84**	−2.24**	0.61**	−2.08**	1.07**	−2.23**	1.01**
	(0.26)	(0.19)	(0.21)	(0.12)	(0.22)	(0.12)	(0.22)	(0.11)
β^{3r}	−2.30**	0.99**	−2.71**	0.57**	−2.25**	1.09**	−2.28**	0..94**
	(0.28)	(0.16)	(0.23)	(0.12)	(0.23)	(0.11)	(0.22)	(0.11)
β^{4r}	−2.18**	1.03**	−2.55**	0.87**	−2.33**	1.25**	−2.47**	1.11**
	(0.28)	(0.16)	(0.22)	(0.10)	(0.23)	(0.10)	(0.23)	(0.09)
β^{5r}	−1.89**	0.97**	−2.55**	0.89**	−2.32**	1.32**	−2.36**	1.18**
	(0.27)	(0.15)	(0.23)	(0.11)	(0.22)	(0.10)	(0.22)	(0.09)
β^{6r}	−1.83**	2.00**	−2.90**	2.44**	−3.10**	2.58**	−3.09**	2.47**
	(0.49)	(0.32)	(0.41)	(0.27)	(0.35)	(0.18)	(0.34)	(0.19)
β^{7r}	−3.26**	2.42**	−3.18**	2.38**	−2.50**	2.16**	−2.73**	2.03**
	(0.78)	(0.71)	(0.72)	(0.64)	(0.48)	(0.31)	(0.49)	(0.34)
β^{8r}	−1.72*	3.26**	−2.60**	3.13**	−1.31**	2.78**	−1.32**	2.47**
	(0.81)	(0.72)	(0.77)	(0.49)	(0.46)	(0.30)	(0.45)	(0.30)
β^{9r}	−7.45**	−1.08	−4.36**	−0.12	−4.26**	0.50	−4.68**	0.31
	(1.18)	(0.75)	(1.04)	(0.63)	(0.86)	(0.43)	(0.81)	(0.45)
β^{10r}	−2.74**	1.01*	−5.36**	−0.04	−3.58**	0.70*	−2.60**	0.50
	(1.01)	(0.49)	(0.89)	(0.47)	(0.74)	(0.31)	(0.65)	(0.31)
β^{11r}	−3.62**	1.11**	−3.39**	1.45**	−2.97**	1.68**	−2.50**	1.70**
	(0.82)	(0.48)	(0.71)	(0.45)	(0.67)	(0.32)	(0.61)	(0.32)
β^{12r}	−4.71**	0.33	−4.47**	1.64**	−3.98**	1.49**	−4.09**	1.20**
	(0.97)	(0.55)	(0.83)	(0.44)	(0.70)	(0.35)	(0.70)	(0.37)
Grade-race-year-ZIP code FEs?	No		Yes		Yes		Yes	
Grade-race-year-ZIP-grade range FEs?	Yes		No		No		No	
R^2	0.36		0.40		0.35		0.33	
Num. obs.	361,866		268,672		344,814		323,784	

Notes: The dependent variable is log enrollment by grade, race, school and year $\left(\log n_{jt}^{gr}\right)$ and specifications include $\log n_{jt-1}^{\bar{g}_j W}, \ldots, \log n_{jt-1}^{\bar{g}_j -1W}$ and $\log n_{jt-1}^{\bar{g}_j M}, \ldots, \log n_{jt-1}^{\bar{g}_j -1M}$ as controls. Specification (2) is estimated on a restricted subsample of school-years that offer instruction in the same grades as in the previous year. Specifications (3) and (4) are estimated on restricted subsamples that exclude all observations for which fewer than 10 and 30 students are enrolled in a given grade at a given school in a given year respectively. Robust standard errors clustered by grade, race, year and ZIP code are provided in parentheses.

* Statistically significant at the 95% level.

** Statistically significant at the 99% level

B.1.1. Simulation

In the empirical model above, s_{jt-1} enters linearly into parental demand functions. This assumption is not overly restrictive in the sense that it does not imply the existence or location of tipping points or the locations of stable equilibria, but it does constrain the shape of S_{jt}. We can modify Eq. (3) by specifying demand as

$$\log n_{jt}^{gr} = \beta^{gr}\left(s_{jt-1}\right) + \alpha_{zt}^{gr} + f\left(n_{jt-1}^{-\bar{g}_j W}, n_{jt-1}^{-\bar{g}_j M}\right) + \epsilon_{jt}^{gr} \tag{B.1}$$

where $\beta^{gr}(\cdot)$ is now a flexibly specified function of the social amenity. The flexible demand Eq. (B.1) can be estimated using appropriate non-parametric techniques (Pagan and Ullah, 1999; Newey and Powell, 2003) to allow for multiple tipping points (and hence more than two stable equilibria) or one-sided tipping behavior (Card et al., 2008b).

We re-estimate parental demand functions by flexibly specifying $\beta^{gr}(\cdot)$ as a cubic B-spline (also known as "natural spline") with knots located at $s = 0, 0.25, 0.5, 0.75, \text{and} 1$. Identification is made possible by the substantial within- and across-school variation in minority share over our sample period.

In Fig. B.16 we present simulated S curves using the flexible demand specification which we overlay on our baseline curves. The curves are qualitatively similar in that they feature the same number of tipping points and stable equilibria. The flexible specification

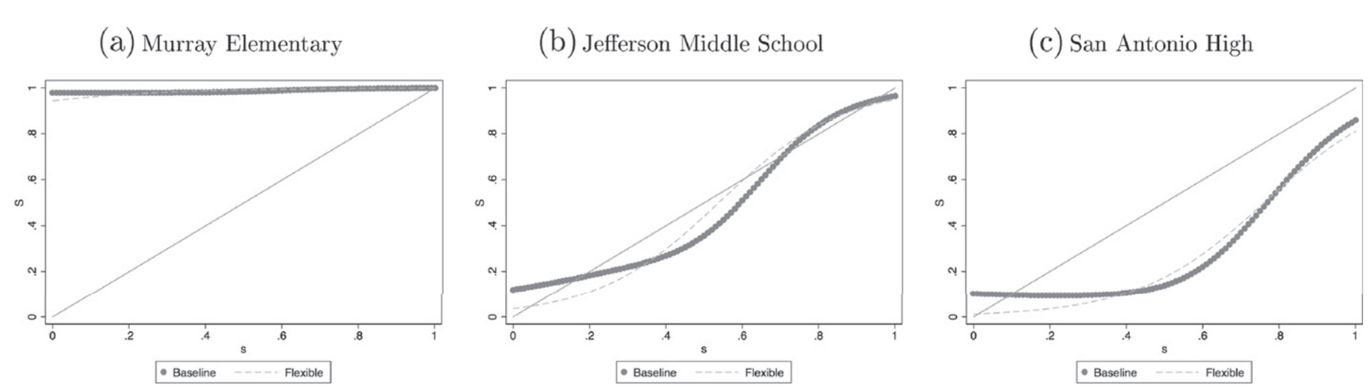

Fig. B.16. Tipping diagrams, flexible demand specification: Three schools. (a) Murray Elementary. (b) Jefferson Middle School. (c) San Antonio High. Note: Each panel plots the S curve for a specific school in 2009. These panels can be compared to the panels of Fig. 5, which use a more restrictive linear specification.

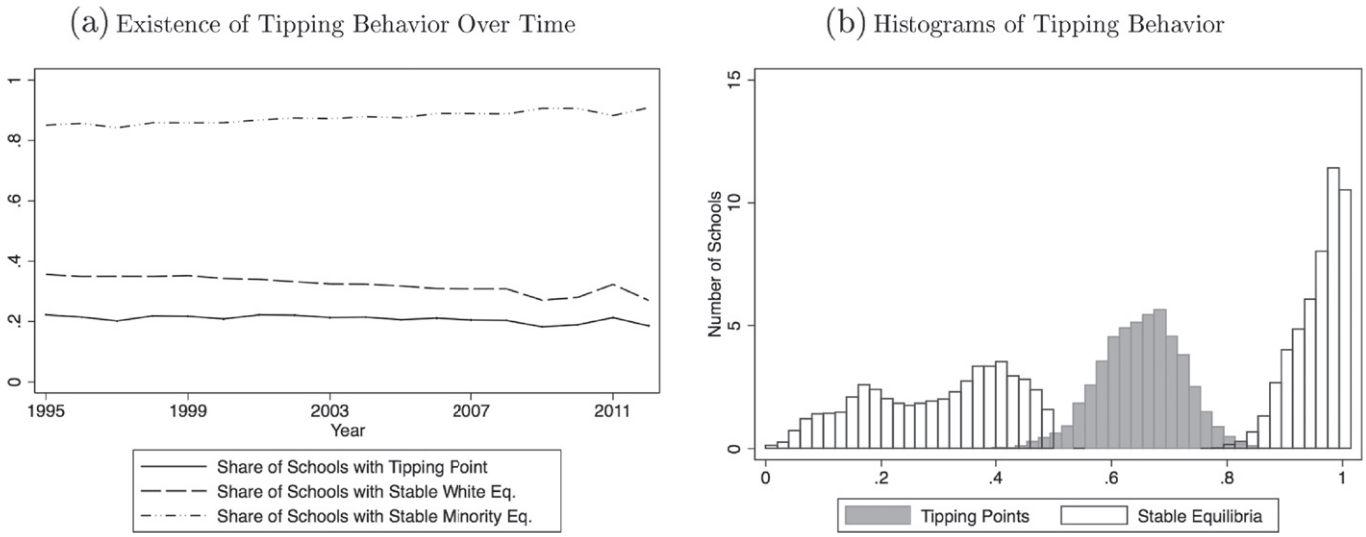

Fig. B.17. Tipping behavior: All schools, flexible demand specification, 1995–2012. (a) Existence of tipping behavior over time. (b) Histograms of tipping behavior. Note: Panel (a) shows the share of schools that possess a tipping point, a stable White ($s < 0.5$) equilibrium and a stable minority ($s > 0.5$) equilibrium for each year. Panel (b) overlays histograms of tipping points and stable equilibria across all school-year combinations in the sample. These panels can be compared to the panels of Fig. 6, which use a more restrictive linear specification.

yields a higher level of implied minority share at lower levels of s, which results in more integrated White stable equilibria and tipping points at higher levels of minority share if they exist at all. We summarize the results for our entire sample in Fig. 6. The existence of tipping points and stable equilibria in the first panel is similar to our baseline estimates, but the locations of tipping points and stable equilibria in the second panel have shifted to slightly higher levels of minority share (Fig. B.17).

Appendix C. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.jpubeco.2017.02.009.

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